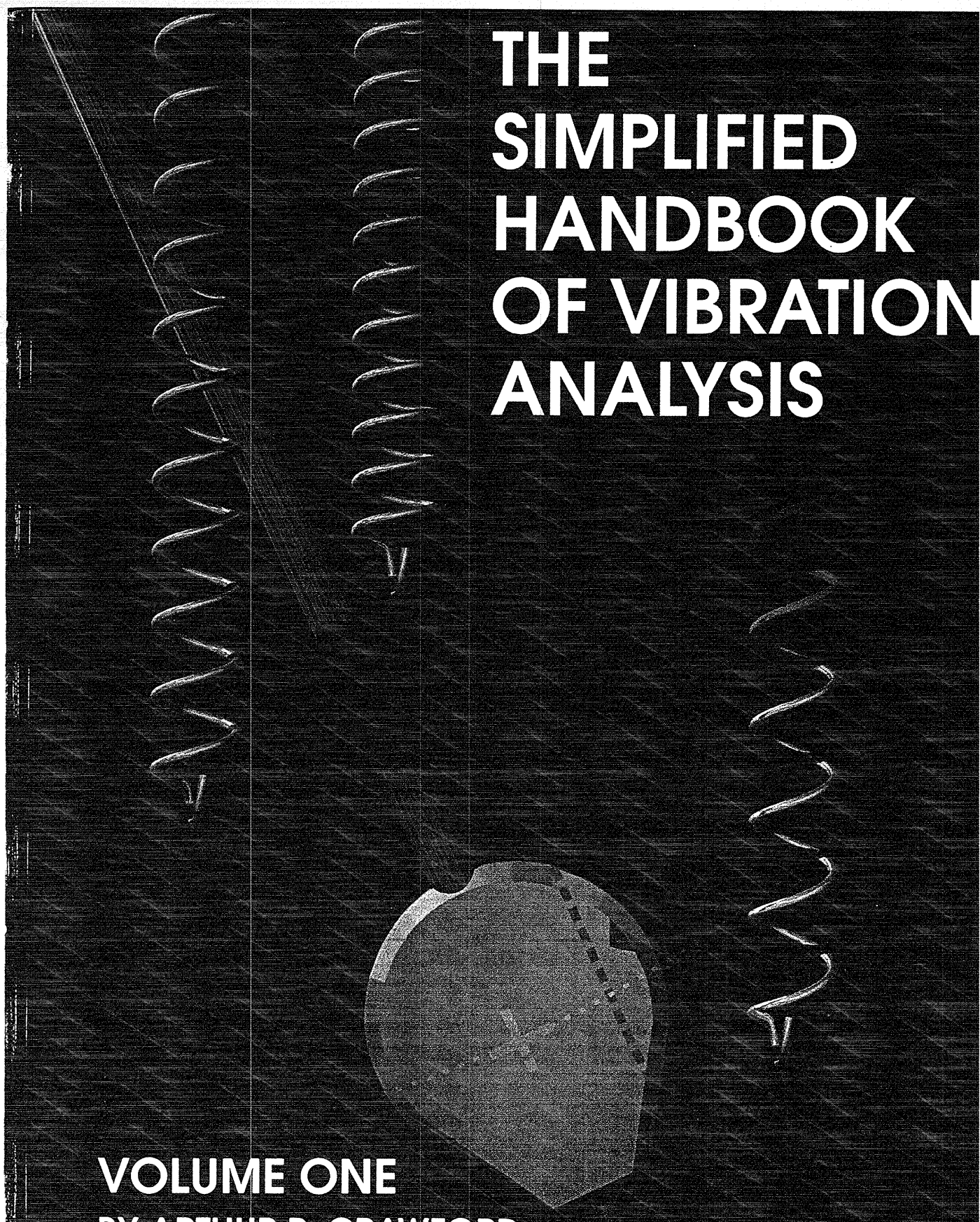
THE SIMPLIFIED HANDBOOK OF VIBRATION ANALYSIS



VOLUME ONE

BY ARTHUR R. CRAWFORD

THE SIMPLIFIED HANDBOOK OF VIBRATION ANALYSIS

Volume 1: Introduction to Vibration Analysis Fundamentals

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The Simplified Handbook of Vibration Analysis
Volume 1: Introduction to Vibration Analysis Fundamentals

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PREFACE

It is difficult to describe the emotion we all felt when we discovered that pure carbon would support free neutrons. Years ago, as a junior research assistant with the Manhattan Project, I was studying methods to enrich nuclear material for use in atomic weapons. At the time the most efficient enrichment methods known were electromagnetic separation and gaseous diffusion.

These two methods, although effective in producing enriched uranium products, were not suited for large-scale production of uranium 235. Early electromagnetic separation efforts only produced one gram of U-235 in an 8-hour day! We needed to build a breeder reactor, where a controlled chain reaction could produce large quantities of enriched material.

A breeder reactor requires a substance that will hold the many free neutrons liberated during the chain reaction. One well known "neutron friendly" medium was deuterium (heavy water), but a check of all the Allied nations yielded slightly more than 2 liters; far too small an amount to build a reactor.

Army intelligence discovered that Germany had over 1,800 liters of deuterium and was producing more every day at the Norsk Hydro plant on Lake Tinnsjö in the Telemark region of southern Norway. It was clear that our enemies were well on their way to building a breeder reactor. This information was so alarming that a suicide mission was sent to destroy the plant. A team of six Norwegian soldiers parachuted into the region on February 16, 1943. Entering the virtually unguarded plant, they succeeded in destroying all 18 electrolysis cells, dumping nearly half a ton of heavy water down the drains without any loss of life on either side. This mission was glamorized and made into a movie (The Heroes of Telemark, Great Britain, 1965, starring Kirk Douglas and Richard Harris).

But the Norsk Hydro plant was not irreparably destroyed. German scientists shipped reserves of deuterium to the plant to refill the electrolysis cells, and the plant began to produce deuterium again in April. On November 16, 1943, the American Eighth Air Force attempted to destroy the plant in a precision bombing strike (a second commando raid was out of the question, since the Nazis had increased security around the plant). None of the bombs hit the plant, but the plant's electrical and hydrogen supplies were damaged. The Nazis then decided to rebuild the facility in Germany, and made plans to move the plant's reserves of deuterium.

Knut Haukelid, the leader of the commando raid in 1943, was given the task of destroying the remaining deuterium before it reached Germany. After careful investigation he decided that the best option was to sink the ferry that would transport the deuterium across Lake Tinnsjö. Late at night on February 20, 1944, Haukelid and Rolf Sörlie planted 19 pounds of high explosives in the hold of the ferry. The next day the charges blew out the hull of the ferry in the middle of the lake. The ferry sank quickly, taking 39 drums (162 gallons) of heavy water to the bottom. Twenty-six passengers and crew were killed.

When we discovered that pure carbon was an acceptable medium, and we no longer needed heavy water, the possibility of a breeder reactor was finally within our grasp. Now all we needed was pure carbon, free of any chemical impurities. One way we found to purify the carbon was to pass high electric currents through it. With that thought we checked the purity of spent electric furnace electrodes, and found to our delight that they were pure enough to support free neutrons.

A project was begun immediately to break up and powder these spent electrodes. A large rock crusher was installed at one of the major carbon facilities in the United States. But, the maintenance supervisor at the plant would not allow us to grind any carbon, because he felt the vibration levels in the crusher were too high.

At that time I knew very little about vibration; I was a nuclear physicist. It seemed to me that the unit was running fine. I called the manufacturer of the rock crusher and gave them the readings provided by the maintenance supervisor. The manufacturer chuckled and replied "with vibrations that low, it must be one of the best rock crushers we ever made!"

When I mentioned this to the maintenance supervisor, he pointed out to me that we were dealing with his plant, not the crusher manufacturer's plant, and he felt that the vibrations were too high. After much debate he finally agreed that we could lower the gate that would permit us to slide carbon electrodes into the crusher. But there was one provision: he would allow us to grind carbon only if the crusher ran with the gate lowered for 24 hours without any trouble.

I was ecstatic. Within one day we would be able to produce powdered carbon, form it into blocks, and build our first breeder reactor. We lowered the gate - a piece of steel 2 inches thick, 4 feet wide, and 30 feet long. The unit vibrated, I felt a little tingling in my feet, but I thought that it was running fine. At 7:00 that evening I returned to my motel, satisfied that there would be no mishaps.

I was very excited and had trouble sleeping, so I returned to the plant early next morning about 6:00. I realized that the 24 hours wasn't up yet, but I wanted to see how the unit was running.

Sometime during the night the gate had sheared off and gone into the crusher. Pieces of steel too heavy to lift had been thrown around the plant. If it had happened during peak hours some people would have been killed; certainly many would have been injured.

To the maintenance supervisor's credit he never said "I told you so," and he pitched in to help us young engineers repair the crusher and return it to service. Once we turned it on it was apparent that the vibration was now severe; so bad in fact that he wouldn't let us run it at all. My superior turned to me and said simply, "balance it."

I'd never balanced anything in my life. Looking back on that balancing job I realize that it was not difficult. I could do it today in one or two balancing runs; it would be no problem at all, especially with a computer. But 50 years ago it was a different story. I spent over a month trying to figure out how to balance a rotor in its own bearings, what kind of equipment I needed to take readings, and what to do with the vectors.

But I finally balanced it. When I finished the floor no longer tingled beneath your feet. Now the maintenance supervisor had no problem with our grinding carbon, and so we prepared to grind our

first electrode. I remember vividly standing there with a long carbon electrode on the gate, ready to push the button. Everyone yelled "Wait!" and ran away. I couldn't understand why.

I pushed the button. I was proud of my machine! That 30-foot long piece of carbon, 3 feet in diameter, was digested instantly by the rock crusher. A cloud of carbon billowed out of the pit, black as night. I brushed it out of my hair, my shirt, off of my neck, out of my eyebrows. It was a messy process, but it ground carbon!

I decided then that when the war was over, when I was away from this responsibility, that I was going to go into the vibration business. For over 40 years it has been a fascinating experience. I've worked on every conceivable piece of equipment in almost every imaginable environment. The technology has been good to me. It has not only supported me and my family, but it has provided a challenge, an excitement that has never varied for over 40 years.

Time and again I've been asked to write a book. I look at textbooks and am positive that I couldn't do any better than that. Even today I read Den Hartog's *Mechanical Vibrations* and am convinced that he was way ahead of his time. There was no reason for me to write a textbook to add to those that already existed.

I was more interested in the solution of vibration problems in industry than I was in writing books about the theoretical concepts behind vibrating structures. I have been asked on many occasions to write a simpler book, addressed to those people who are doing the work, whose primary interest is in solving the vibration problem and correcting the equipment.

And that is what we've tried to do. I have enjoyed working with my son, and I admit that he has shouldered the major portion of the writing. I've cringed at times because some portions of the book seem so non-technical, and yet in the main I feel that it is a simple book that deals with some complex subjects. And though I have no grand thoughts that we have contributed much to the technology, I hope that there will be interest and enjoyment in reading a simple approach to vibration analysis and vibration problem solving.

I would like to extend my heartfelt thanks to Dr. Kenneth Piety for pushing me into this venture, to my son for his pushing and prodding to get the work done and for his wonderful self-control in working with his father. I also want to include all those who took the time to help us in writing, reviewing, and correcting this book. And, my deepest appreciation goes to my loving wife, who not only patiently endured with me during the difficulties and hardships of starting a new business, but has also wholeheartedly supported me over the years as well as in this effort.

Art Crawford

I can honestly say that I have been involved with the vibration industry for most of my life. While other kids were playing baseball with their dads, I was winding transducer coils and building strobe lights in our basement. My father never took me fishing; he took me to paper mills, automobile assembly plants, coal mines, power plants, petroleum refineries, and factories of every size and description.

And I loved every minute of it. I've climbed over turbines, straddled I-beams on the 40th floor of empty skyscrapers, and stood on cooling towers overlooking the Dead Sea. It has been a tremendous experience. I do not share my father's concerns that this book is too non-technical. Rather, I fear that we have been unable to convey the excitement of the detective work, of isolating and correcting a vibration problem.

We have tried very hard to keep our discussion of vibration analysis as simple and non-mathematical as possible. But talking about vibration analysis without math is akin to discussing

the Kentucky Derby without mentioning horses. At several points in the text we were forced to decide whether or not to omit a more detailed discussion of a topic due to the math involved. So these discussions are set apart from the main text in gray boxes; if you would like additional information, then read the text in the box. You may skip over the gray boxes if you prefer.

There are many people whom I would like to thank for their help during the completion of this first volume. Brian & Nancy Bauerbach and Wes Page provided many valuable comments during the technical review of the first draft, along with Ken Piety, Forrest Pardue, Dan Nower, and Ray Garvey at CSI. Jenny Hecker did an excellent job editing the final draft, ensuring both correctness of English and consistency of style. Kurt Vonnegut advises that an author should never index their own work. With that in mind, Gail Emelko did a wonderful job coding the entire volume to produce the exhaustive index in the rear of the book. Jack Seibert and his wonderful staff (the world's greatest jeweler!) provided a great deal of information on tourmaline and other piezoelectric crystals, and showed me lots of pretty samples as well.

I would also like to thank Dr. Franklin Miller for allowing us to print the photographs of the Tacoma Narrows Bridge collapse. Dr. Miller was also very helpful in providing technical papers on the bridge collapse. Many additional thanks to Ken Piety for his support and patience during this project (which went on a *little* longer than I thought it would). And I would also like to thank Ron Moore at CSI for making the entire project possible.

Finally, and most importantly, I must thank my wife Ann. She has been a comfort, a sounding board, a source of infinite wisdom, and the greatest inspiration a writer could have. Our son Adam was born during the development of this book, and I would like to dedicate it to him. Welcome, son!

Steve Crawford

ABOUT THE AUTHORS

Art Crawford has been an active consultant in the vibration industry for the last 45 years. He is the Founder and President of ARC Associates, a vibration consulting firm with customers worldwide, and is a senior vibration analyst for Computational Systems, Incorporated (where he also teaches courses in advanced vibration analysis).

In 1969 Crawford founded Production Measurements Corporation (PMC), a leading manufacturer of electronic equipment for engine analysis, vibration analysis, and vibration monitoring. Prior to 1969 he was director of electronic research for ABEX Corporation, where he was retained as a consultant on the design and development of engine vibration analysis and dynamic balance equipment for the automotive industry.

Art was the Founder and President of IRD Mechanalysis in 1952. IRD, profitable every year under his direction, remains a leader in the field of vibration analysis and dynamic balance equipment. He holds a B.S. and M.S. in Physics from The Ohio State University.

Steve Crawford is the Founder and President of Crawford Communications, a technical publication consulting firm in Columbus, Ohio. He holds a Bachelor of Arts from The Ohio State University, much to the chagrin of his father.

1 INTRODUCTION TO VIBRATION

Of all the parameters that can be measured in industry today, none contains as much information as the vibration signature. Even though proper maintenance of industrial equipment may require the measuring or monitoring of other parameters such as

- temperature
- pressure
- flow
- voltage
- current
- horsepower
- torque

the fact remains that the vibration signature contains more information about the machine's health and operating characteristics than any other parameter.

That is not to say that we are presently able to extract all of the information contained in that vibration signature as we attempt to analyze a piece of equipment. However, our ability to extract and analyze information from vibration data has grown dramatically in the last 15 to 20 years.

EARLY METHODS OF VIBRATION ANALYSIS

No more than a generation ago, the quality of operating equipment from a vibration standpoint was determined by using a coin on edge or feeling the bearings of the machine. If the coin remained

vertical while the machine was running or the bearings felt good to the inspector, the vibration was assumed to be acceptable. On the other hand, if the coin fell over or the bearings felt bad, an attempt was made to balance some part of the machine.

Plant personnel quite often concluded that a machine vibrated because it was out of balance; other causes of machine vibration (such as misalignment, bent shaft, hydraulic instability, or bearing failure) were rarely suspected. Large machine tool manufacturers would ship what was considered to be high-quality machinery, where the only test for vibration was how the machine "felt" to the quality control inspector or manager.

Rotors were balanced by using chalk or soapstone held lightly against the shaft. A mechanic would look at the arc of the chalk mark and estimate the magnitude and location of the imbalance. Mechanics would also hold a pointer close to a shaft and, by eye, judge the magnitude of the vibration by watching the varying gap between the pointer and the shaft.

As frequencies increased, it became much more difficult to estimate the amount of vibration. It was not uncommon to mount dial indicators against a bearing housing or a shaft, and to use the dial indicator to read the vibration amplitude. As shaft frequencies increased the mass integration of the dial indicator made it impossible to read the displacement accurately, if at all. That is, for a given input energy, a mass has greater displacement at lower frequencies than at high frequencies. So the mass of the dial indicator moves less at high frequencies than at low frequencies, and is therefore less sensitive at these higher frequencies.

A light beam vibrometer was developed which had very low mass integration. This device could read vibration amplitudes at frequencies as high as 2,000 CPM (cycles per minute) without degradation, and as high as 4,000 CPM with some loss of reliability and accuracy.

ELECTRONICS IN VIBRATION ANALYSIS

All of these early instruments measured displacement: the space occupied by the vibrating system. When electronic equipment was developed to work with vibration transducers, it was natural to read the amplitude in units that were already common. So the early amplitude meters read vibration in mils peak to peak (1 mil = .001 inch).

Survey programs were established where overall vibration readings were taken and recorded on data sheets carried by the operator. Even though the trend of the overall vibration level was much better than no data, it was of little use in pinpointing the source of the vibration and did not lend itself to post-failure analysis.

It soon became apparent that since vibration was usually caused by some component in the mechanical system, frequency analysis would be a powerful aid in analyzing machinery vibration and identifying the forcing function.

Early analog analyzers were developed which consisted of

- an amplitude meter
- a frequency meter
- a tunable constant percent bandwidth filter
- a strobe light for phase measurement

The tunable filter made it possible to separate various vibration frequencies and to identify these frequencies with machine components. The time required for frequency analysis was long, and frequency resolution and accuracy were poor due to circuitry design constraints and the difficulty in

manually tuning a very sharp analog filter. But frequency analysis proved to be a valuable and informative analysis tool, and these analog analyzers are still in common use today.

In addition to their elementary analysis capability the analog analyzers could be used to field balance equipment. In fact, this feature alone led to their early acceptance in industry. Although the theories behind reading relative phase with a strobe light and allowance for cross effect between planes were not understood by most of those who used the instrument, field balance by single plane iteration became quite common.

One unfortunate consequence of the analog analyzer was that it led people to ignore the value of the time domain signals. Most of these analyzers included an output that could be connected to an oscilloscope so that time waveforms could be analyzed, but virtually no one used an oscilloscope with those vibration analyzers.

DIGITAL ANALYSIS OF VIBRATION SIGNATURES

Serious development of digital sampling techniques were initiated during World War II and the mathematics were quite well understood in the decade of the 1960's. Although the electronic hardware was not economical, the advantages of these techniques were obvious. The digital system

- is more stable
- is more linear
- has a greater dynamic range
- is more accurate, especially in the frequency domain
- is inherently a true RMS system (RMS is defined on page 19)
- is more flexible
- is real time (at least at the lower frequencies)
- functions as a constant bandwidth filter
- has no physical inertia
- is ideal for transient data capture
- lends itself to other mathematical manipulations of vibration data such as coherence, transfer function, phase, and time domain averaging. These are explained in detail in the Digital Analysis chapter in Volume II.

The first digital analyzers were very expensive, relatively complicated to run, and heavy. It was therefore natural to conclude that analog analyzers were the best approach for industrial vibration analysis, and digital analyzers were best for advanced laboratory research and military applications.

With the advent of the personal computer, the components necessary for digital analysis have improved in performance and dropped dramatically in price. Whereas the analog vibration analyzers (though virtually unchanged in performance) have increased in cost 5 to 10 times over the last generation, digital analyzers have reduced in cost 5 to 10 times over the same period.

Digital analyzers have become "user friendly", low in cost, lightweight, and extremely capable in routine data collection. The analysis of vibration data in both the time and frequency domains is greatly facilitated by digital analyzers. In addition the data storage, data manipulation, post processing, and hardcopy output capabilities are far beyond those of the older analog instruments.

There have also been dramatic developments in vibration transducers. Early pick-ups, as many called them, were designed as displacement transducers. One innovative example was the Mechano-

tube, a vacuum tube which had the grid connected to a flexible diaphragm; the gain of the tube was changed in a linear manner as the grid post was deflected.

Many types of transducers have been developed. Some examples are carbon piles, variable resistors, capacitive or inductive arrangements, conducting rubber, linear variable differential transformers, strain gauges, and eddy current probes. The eddy current proximity probe has had the most dramatic development of all of these devices and is usually the transducer of choice for displacement output.

As interest in vibration analysis techniques increased, the academic community introduced more vibration theory into the engineering curriculum. As theory meshed with practice it became apparent that velocity was the better parameter to indicate vibration severity and that acceleration was the better parameter for high frequency analysis.

A surge of velocity transducer development resulted in several seismic versions which are still in common use. Although these units worked fairly well they were quite heavy, had a limited frequency and temperature range, and were rather high priced. There is little or no development work in this area today.

The greatest strides have been made in the design and development of the accelerometer. Highly stable quartz units have been developed with the charge amplifier built into the transducer to reduce the lead length, lead configuration, and tribo-electric effect (electric noise generated by flexing a high impedance transducer cable; explained in detail in the Transducers chapter). Some of these units have output impedances lower than 100 ohms so that unshielded cable can be used.

If the improvement in instrumentation, transducers, and application continues to grow at the present rate, vibration data collection and analysis will be the most important factor in maintenance and factory throughput.

The capabilities of modern digital analyzers and the value of the information contained in the vibration data make it imperative that both management and maintenance understand the basics of vibration and the application of vibration analysis to the production process.

USES OF VIBRATION ANALYSIS

The collection and reduction of vibration data allows you to perform three important analysis techniques:

1. Certification
2. Potential fault analysis
3. Diagnostic analysis

Certification

Vibration data is taken on new and/or rebuilt machinery to ensure that the machinery is operating within acceptable vibration tolerances. If excessive vibration is discovered, the cause can be remedied before the machinery is put into service.

Potential Fault Analysis

Also called "predictive" maintenance, potential fault analysis allows you to determine the health of machine components. A good assessment of machine health allows you to predict what component is most likely to fail first, and approximately how long that component will continue to operate before

fault conditions begin to occur. This information allows you to anticipate and schedule machine overhauls, thereby minimizing the impact of process shutdowns and avoiding damage that results from running a machine until a catastrophic failure occurs.

In potential fault analysis, information about the machine is used to determine the number and location of data collection points. Machine information includes

- the number, type, and location of each bearing
- the type and location of couplings
- machine components (turbine, pump, generator, etc.)

These topics are discussed in detail in subsequent chapters.

Mathematical techniques (described in later chapters) are then used to calculate approximate fault frequencies for each system component. If possible, the phase relationships of component faults can also be determined.

Possible causes for each fault frequency can also be determined, and each fault frequency is prioritized. For example, probability of each fault is determined, the seriousness of that fault is assessed including the cost of down time and repair, and appropriate collection intervals are scheduled.

The vibration data is analyzed to determine if any of the fault frequencies are present and, if so, their severity. Vibration tolerances, set in conjunction with manufacturer's specifications, are used to determine severity. Proximity to an alarm condition indicates the most likely fault and the trend allows you to approximate when it will occur.

A carefully maintained potential fault analysis program, coupled with a program of machine maintenance, ensures long, trouble-free operation of plant machinery as well as better product quality.

Diagnostic Analysis

Whereas potential fault analysis is more proactive, diagnostic analysis is reactive: it is only performed once a problem has occurred. Often times the problem will be so severe that equipment must be shut down for safety reasons.

Diagnosis begins with the same steps as potential fault analysis. Machine data is used to determine the number and location of data collection points, and fault frequencies and phase relationships are calculated.

Once the vibration data is collected, a comparison is made between the calculated fault frequencies and their harmonics and the actual frequencies that dominate the vibration signature. The cause of each fault frequency is determined, and corrective measures are taken to correct the problem. Follow-up readings are taken to ensure that the problem was indeed solved and that the machine is operating within tolerance.

If a potential fault analysis program is not already in place, then the best time to begin the program is immediately after a diagnostic analysis. The machine data, fault frequencies, and possible causes have been determined. An inspection of the machine will have been completed during the diagnosis to ensure that no other causes of vibration are present, such as

- loose bolts
- cracked weldments
- grouting turned to powder
- ash accumulation on fan blades

Now that a thorough inspection of the machine has taken place, the machine is operating smoothly and the current state of the machine is fresh in the operator's and maintenance manager's mind. A potential fault analysis program should be implemented now to ensure that the machine remains in good health.

This handbook discusses vibration, vibrating systems, transducers, instrumentation, and analysis techniques in both the time and frequency domains in an effort to give you a sound foundation upon which to build your knowledge and experience in this exciting field.

2 SYSTEM RESPONSE

The field of vibration analysis deals with the behavior of vibrating systems. You no doubt have your own idea as to what a system is: a steam turbine, an AC generator, an electric motor, a pump, and so on. The purpose of this section is to describe how *any* system behaves when it vibrates. Therefore, we need a definition of a "system" that is clearer and more precise than any individual connotation. As we progress through this chapter, you will see that our definition of a system is precise enough for any application.

As we define a system and explain how systems vibrate, we will discuss five "rules" that describe how all systems respond to vibratory force.

1.0 A MECHANICAL SYSTEM

A **mechanical system** consists of three main components:

1. mass
2. spring
3. damping

1.1 Mass

In physics, **mass** is defined as "a body's resistance to acceleration." In everyday terms, a mass is a body of matter that occupies a certain volume of three-dimensional space. This definition is true regardless of the shape of the object or of the material with which it is made. This paper is a mass; so is the chair that you are no doubt sitting on as you read this; and so, in fact, are you. All three of these objects are different shapes, and are made of different materials, but they are all considered a mass.

Do not confuse the idea of mass with the idea of weight. It is certainly true that all the masses that you encounter every day have weight, but mass and weight are two different concepts: the weight of a mass is a result of gravity, whereas mass is a property of the object itself.

Weight is a force that results when a mass is accelerated. On Earth, a 16-pound bowling ball weighs 16 pounds because of the force of gravity on each atom within the ball. On the moon, where the gravity is 1/6 that of Earth, the bowling ball only weighs 2.66 pounds. In the furthest reaches of space, in the absence of any gravity, the ball has no weight at all. The mass of the bowling ball has not changed, because it still occupies the same amount of three-dimensional volume. Only the weight has changed, because the gravity has changed.

In defining a system, mass refers to any part or body within the system that is free to move and is made to move by force. This motive force contributes to the vibrating system's behavior. For example, consider a steam turbine. Certainly there are many "masses" within the turbine; the rotor, the turbine blades, the housing, and so on. But the blades and the housing are held rigidly in place (we hope), and are not forced to move independent of any other part.

The shaft, however, is free to move, and is forced to move by steam impacting the rotor blades. Therefore the rotor and the blades move together, as one mass. The bearings, although free to move internally, are considered a separate mass from the shaft/blade assembly. As you will see in other chapters, excessive vibration in either of the bearings can be transmitted to the shaft. Likewise the shaft can transmit vibration to the bearings. The resolution of vibration such as this will treat the bearing and the shaft as separate masses.

1.2 Spring

A system's **spring** is any part of the system that supports mass and behaves like a spring.

When you think of a spring you no doubt envision a piece of coiled steel that pushes (or pulls) against a force. This pushing or pulling strength of a spring is called the **spring constant**, and is expressed mathematically by the letter k. The spring constant is defined as the number of pounds necessary to extend the spring one inch, and is calculated by the formula

$$k = \frac{F}{x}$$

where

k = the spring constant (lbs/in)
F = force (lbs)
x = amount of displacement (inches)

The "backwards" pull of a spring (against the tension) is called the **restoring force** of the spring.

In the case of our steam turbine, each of the turbine blades bends slightly when the steam hits it. The shaft will want to twist slightly, and the bearings supporting the shaft may also move. Each of these masses is a spring-mass unit because that is exactly how it behaves. The spring constant of each turbine blade determines how far it bends when impacted by the steam. Likewise the spring constant of the shaft determines how far it twists.

The spring constant of each spring-mass in the system determines how much the mass moves in response to a force, and results in a restoring force that contributes to the overall dynamics of the system. We will see the effects of this restoring force later.

1.3 Damping

In order for motion to occur, a force must be applied to a mass. But the mass will not move forever; something always dissipates the energy and causes the mass to come to rest.

The process of energy dissipation is called **damping**. There are three main forms of damping:

1. Friction damping
2. Viscous damping
3. Hysteresis damping

1.3.1 Friction Damping

Friction damping occurs when two parts of a system slide against each other. If you slide your hand rapidly across a table, a friction force is exerted against your hand. This force prevents your hand from sliding easily, and dissipates some of the energy of motion of your hand. Since energy must be conserved, both your hand and the table get warmer. The friction force between surfaces is the product of two quantities: the coefficient of friction and the load.

The coefficient of friction is a constant that expresses the amount of friction between two surfaces. This constant depends, among other things, on the physical properties of the two surfaces. There is a very low coefficient of friction between ice and steel; a cube of ice slides very easily across a flat plate of steel. A block of rubber, on the other hand, would not slide very easily across a steel plate since the coefficient of friction between rubber and steel is higher.

Two sliding surfaces push against each other with a certain load; the greater the load, the higher the friction force. For example, a certain coefficient of friction exists between a rotating brake drum and a brake shoe. Since this coefficient is a constant at constant temperature, only the load will cause the friction force between the two surfaces to vary.

If the brake is not applied, the two surfaces slide against each other with a fairly small friction force. However, when the brake is applied, the brake shoe pushes hard against the brake drum. This increased load dramatically increases the friction force and causes the drum to stop.

The friction force, and therefore the amount of energy loss due to friction damping, is directly proportional to displacement (movement). If the brake drum were not moving, the friction force would be zero regardless of the load.

Friction forces are usually very low in industry, so the effect of friction damping on vibration is usually negligible. Bearings and other sliding surfaces are designed to minimize friction by using lubricants. The introduction of lubricants to reduce friction damping, however, creates viscous damping.

1.3.2 Viscous Damping

Viscous damping results from the resistive force against a body moving through a viscous fluid. A shock absorber in an automobile is a common example of a system with viscous damping. The term "viscous" simply means having a resistance to flow. The more viscous a liquid, the thicker it is and the more resistant it is to flow. For example, water pours from a jar much more easily than molasses, since molasses is more viscous than water.

In order for an object to move through a viscous fluid, the object must push the fluid out of the way as it moves. Since the object is pushing against the fluid, the fluid exerts a resistive force against the object. This resistive force is determined by four quantities: the size of the object, the size of the fluid's container, the viscosity of the fluid, and the velocity of the object.

The physical dimensions of both the object and the fluid container are constants. The viscosity of the fluid can also be considered a constant provided that no extreme temperature fluctuations are present, since fluids tend to thicken as they get colder. Therefore the only variable in calculating the resistive force (and therefore the amount of energy lost to viscous damping) is the velocity of the object: the higher the velocity, the greater the resistive force. If the velocity is zero then the resistive force is zero.

Imagine a bicycle pump. The handle is connected to a plunger that moves down the cylinder. As the plunger moves it pushes air (which is a viscous fluid) out of the tiny hole at the other end of the pump. If you push the handle in slowly it moves fairly easily since the amount of air that needs to flow out of the hole in unit time is rather small. Now if you try to push the handle in very fast, you encounter a greater resistive force. At the greater velocity much more air needs to move out of the hole in the same unit time.

Viscous damping is the major source of damping found in industry, and has two major sources: bearing lubricant and the process itself. Lubricant inside bearings provides a resistive force against the movement of balls and shafts in sleeves. Therefore, any vibration being transmitted to a bearing is also damped by the fluid's resistive force. If this damping is enough, little or no vibration is transmitted to other parts of the system.

Most processes in industry are viscous processes: pumping air, pumping sewage, grinding pulp (air is considered a fluid since its dynamic behavior is mathematically identical to fluids). All these are viscous processes, since they deal with fluids. And since the viscosity of a fluid contributes to the resistive force, the fluid in the process will provide viscous damping.

Generating electricity is also a viscous process: the faster a turbine spins, the greater the voltage out of the generator. The resultant current exerts a "back pressure" against the turbine which is essentially the same as viscous damping. In fact, you will often hear vibratory damping referred to as "Coulomb damping," named in honor of Charles Augustin de Coulomb (1736-1806), a French physicist who studied damping in electrical circuits.

1.3.3 Hysteresis Damping

Hysteresis damping (also called "structural damping") is internal friction that occurs in solid materials, particularly metals. This type of damping is due to the slipping and sliding of internal planes when a material is bent.

When a spring is stretched, or when a shaft or other solid material deflects, internal stresses build up as the molecules slip past each other. This slipping creates friction between the molecules, and this friction force opposes the deflecting force and causes damping. Since this friction occurs at the molecular level, however, its effect is almost unnoticeable. Hysteresis damping is higher in castings than in weldments.

2.0 DEGREES OF FREEDOM

Mechanical systems are designed to move in certain ways, and not to move in other ways. For example, a piston in a cylinder is designed to move only up and down. There are two types of movement: rotation and translation.

Rotation is the turning of a body around a central axis. The wheels on your car, a gyroscope in its frame, and a cooling fan are all examples of rotating systems.

Translation is movement along an axis. A box sliding down a ramp and a piston in a cylinder are translating systems.

In general, then, the number of variables needed to define the position of a mass within a system is the degree of freedom of the system. The position of the piston in Figure 1 can be defined by its position along the y axis (that is, by its distance from the cylinder end). Since it cannot move in the x direction (side to side) or in z (back and forth), and it is not free to rotate, the distance from the end of the cylinder is all that is needed to define the piston's position. Therefore this is a one degree of freedom system.

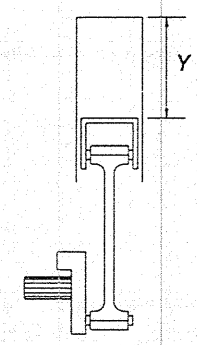


Figure 1 - *a one degree of freedom system (translational)*

A rotor held in bearings like the one in Figure 2 is another example of a one degree of freedom system. Assuming that the bearings are rigid and that they do not move, the position of the shaft can be defined by the angle between a reference point on the shaft and an external reference plane.

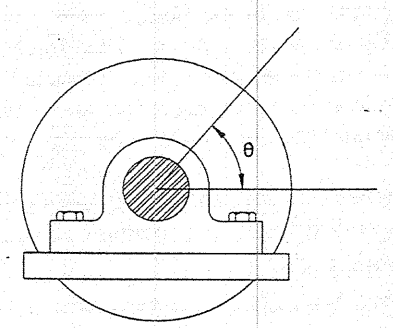


Figure 2 - *a one degree of freedom system (rotational)*

A rigid body moving freely through space has six degrees of freedom: three translations and three rotations. In vibration analysis we are primarily concerned with single degree of freedom systems. For most systems with multiple degrees of freedom, the vibration of greatest concern usually occurs in the region of the first degree of freedom.

3.0 FORCING FUNCTIONS

Mechanical systems do not vibrate all by themselves; something has to force them to vibrate. The simplest vibrating system is a pendulum, and we will look at pendulums quite a bit in this chapter. Think about a grandfather clock: hanging the pendulum and pulling the weights up does not cause

the pendulum to move. Left by itself the pendulum would never move. To start the pendulum moving, you must reach inside and force it to move by pushing it.

This is true for all mechanical systems. Any system that is vibrating is forced to vibrate. A force that causes a system to vibrate is called a **forcing function**. The word "function" refers mathematically to a continuous event that varies with respect to a variable. For example, the contrast of a photographic image is a function that varies with how much light strikes the negative; the more light, the better the contrast.

There are many sources of forcing functions, a few of which include

- imbalance in a rotor
- bearing failure
- shaft misalignment
- a bent shaft
- eccentric gears

In vibration analysis, forcing functions vary with time. We will discuss and illustrate this in detail later on, so for now keep in mind that a vibrating system is forced to vibrate, and that the vibratory force varies over time.

We have now defined a system as having mass, spring, and damping, and have stated that systems are forced to vibrate by a forcing function. Now let's define vibration.

4.0 CHARACTERISTICS OF VIBRATION

A vibrating system is oscillating (moving) in equilibrium about a point. Equilibrium means that, even though the system is oscillating, the vector sum of all the forces acting on the system at any instant add up to zero (vector sums are discussed later in this chapter). If they did not, then the system would not only be vibrating but would also be scooting across the floor. This of course would make it impossible to correct; the translation would first have to be eliminated so that all that remains is vibration.

The simplest vibrating system is the pendulum, and we will use a pendulum to illustrate the terms that define vibration. Like any mechanical system, a pendulum consists of three parts:

1. **mass**: the mass hanging at the end of the pendulum (for now, let's assume that the string holding the mass is weightless. We will treat the mass at the end of the string as the only mass in the system.)
2. **spring**: for a pendulum, the spring is gravity (once the pendulum is set in motion, gravity acts as a restoring force to pull the pendulum back to rest)
3. **damping**: friction at the pivot and air resistance

For the moment we will assume that damping does not exist, and that once the pendulum was set in motion it would continue to vibrate forever. Certainly such a pendulum does not exist; even if we put it in a vacuum to eliminate the air resistance we would still have friction at the pivot and molecular friction (hysteresis damping) in the cable itself. But for right now, pretending that damping does not exist makes vibration much easier to define. We will address damping later in this chapter.

If a pendulum is pulled back and released, it swings back and forth (that is, vibrates) at its natural frequency. In order to describe this vibration, we must first decide on a frame of reference for viewing the behavior.

We will say that the "at rest" position of the pendulum (hanging straight down and not moving) is the 0 position. Movement to the right of the 0 position will be considered positive, and movement to the left of the 0 position will be considered negative. Figure 3 illustrates our frame of reference.

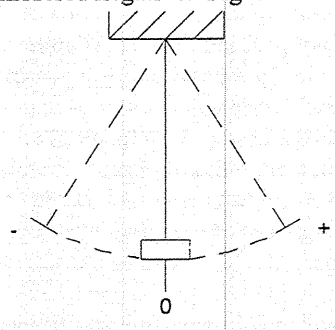


Figure 3 - *viewing pendulum motion*

Remember, this is an arbitrary definition; we could just as easily make the leftmost position zero; then all motion would be in the positive direction and we would have no negative numbers. Most engineers and physicists use a definition similar to ours, with zero in the middle, positive to the right, and negative to the left. Now, given this standard, let's label the drawing (refer to Figure 4) and define some terms.

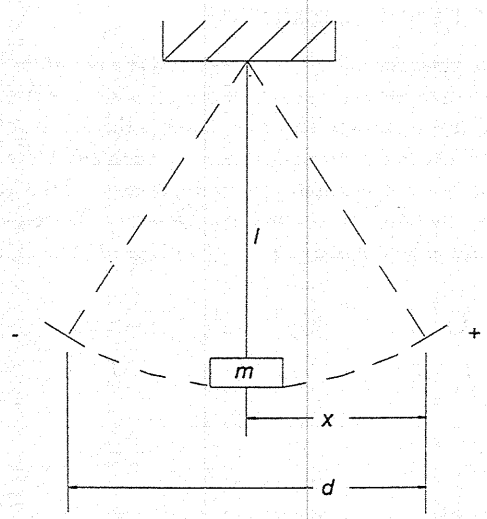


Figure 4 - *defining pendulum motion*

Amplitude (x) - Amplitude is the distance from the "at rest" position. However, amplitude is rarely used as a measure of vibration. If someone uses a dial indicator and says that a shaft is moving 1/4 inch, they're really saying that it's moving back and forth 1/4 inch. English units are mils (1 mil = 1/1000 inch, so 1/4 inch equals 250 mils), and metric units are either millimeters (mm) or microns (1 micron = 10^{-6} meters).

Displacement (*d*) - Displacement is the total movement of the mass. Also called "double amplitude" or "peak-to-peak displacement", displacement equals 2 * *x*. Industry has measured this for years. English units of displacement are mils, and metric units are millimeters (mm).

Velocity (*v*) - Velocity is the measure of change in position during unit time. The velocity of the pendulum is 0 at the farthest negative position. The maximum velocity is at the 0 point, then the pendulum slows down again as it moves toward the farthest positive point. English units of velocity are inches/second (written in/sec), and metric units are mm/sec (sometimes cm/sec).

Acceleration (*a*) - Acceleration is the change in velocity in unit time. The pendulum accelerates when it is moving toward the 0 point from either direction. Acceleration is measured in g's in both the English and metric systems (one *g* equals the acceleration due to gravity - 32 ft/sec or 9.8 m/sec).

Frequency (*f*) - Frequency is the number of complete cycles in unit time. Vibration frequency is usually measured in cycles per second (Hertz) or cycles per minute (CPM) in both the English and metric systems.

A vibrating system is a system which is in equilibrium and has all these parameters (amplitude, velocity, acceleration, and frequency) simultaneously. If you fell out of an airplane at 40,000 feet, you would have acceleration, velocity, and displacement; you wouldn't have any frequency (at least until you hit the ground). Therefore, you would not be a vibrating system.

When you are driving down the freeway at a steady 60 mph, you have both velocity and displacement (you also have amplitude, since displacement equals 2 * amplitude). However, you have no acceleration or frequency. Therefore you are not a vibrating system.

5.0 SINE WAVE PLOT OF PENDULUM MOTION

Imagine now that the entire pendulum (both the mass and the weightless string holding the mass) is made of glass, glass so clean that you can't see it. If we paint a black dot on the bottom of the mass and ask you to lie down under the pendulum, all you will see is the black dot. If we pull the pendulum back and release it, all you will see is the black dot moving back and forth in a straight line.

Now imagine that we mount a spring-loaded pen on the bottom of the glass pendulum. Under the pen we place a roll of paper, so that the pen is touching the paper. If we pull the pendulum back and release it, and pull the paper out at a constant rate, a curve like the one in Figure 5 will be drawn on the paper.

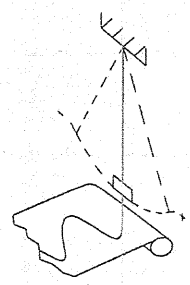


Figure 5 - *plotting pendulum motion*

This curve is called a time domain plot of the motion, since it shows how the vibration varies over time. Now let's make the same assumptions that we made earlier:

- the x (horizontal) axis is 0
- positive is above the 0 line
- negative is below the 0 line

The terms defined from our previous discussion of the pendulum's movement are all contained in the time domain plot, as shown in Figure 6.

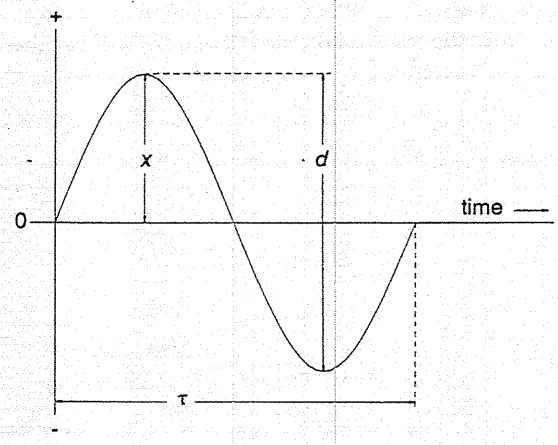


Figure 6 - *labeling the pendulum plot*

x = amplitude - the distance from the 0 line
d = displacement - the total movement of the mass (the double, or peak to peak, amplitude)

Comparing the two pictures of the pendulum, we can see that

- when the pendulum is at the rightmost position, the curve is at the highest point in the positive direction
- when the pendulum is at the leftmost position, the curve is at the lowest point in the negative direction
- when the pendulum is at the 0 position, the curve is on the 0 line

The x axis represents time, because the paper was pulled out at a constant rate. The period of this wave, represented by the Greek letter τ (pronounced "tau"), is the time between two successive equal conditions of motion. For example, the curve in Figure 6 starts at 0 on the x axis. The next 0 value along the x axis may be the next one in succession, but it is not an equal condition of motion. The curve left the first 0 in a positive direction; it leaves the next 0 value in a negative direction. Therefore these are not equal conditions of motion. The end of the period is as shown, when the curve once again leaves the x axis in a positive direction.

Any two successive equal conditions of motion can be used to define the period of the curve. For example, the time between two maximum (or minimum) amplitudes is one period, as is the time between two halfway marks moving in the same direction. If we wanted, we could mark the x axis off in seconds. Adding up the number of marks between two successive equal conditions would tell us the length of a period. Frequency is defined as $1/\tau$. For example, a motor at 1800 rpm goes one

revolution in $1/1800$ of a minute. The period is $1/1800$ of a minute, and the frequency equals $\frac{1}{1/1800}$, which equals 1800 cpm.

Those of you that have taken trigonometry will recognize Figures 5 and 6 as a plot of the sine function; therefore this is also called a **sine wave**. Instead of marking the x axis in time increments, a sine wave marks the x axis in degree increments. One period corresponds to one revolution around a circle, so one period is equal to 360 degrees.

Look at Figure 7. We have a mass at the end of a radius, all ready to rotate around the circle in a counter-clockwise direction. Since the mass hasn't moved yet, we will say that it is at the 0 position. If you imagine the circle as a clock face, you will see that the mass is resting at the 3:00 position.

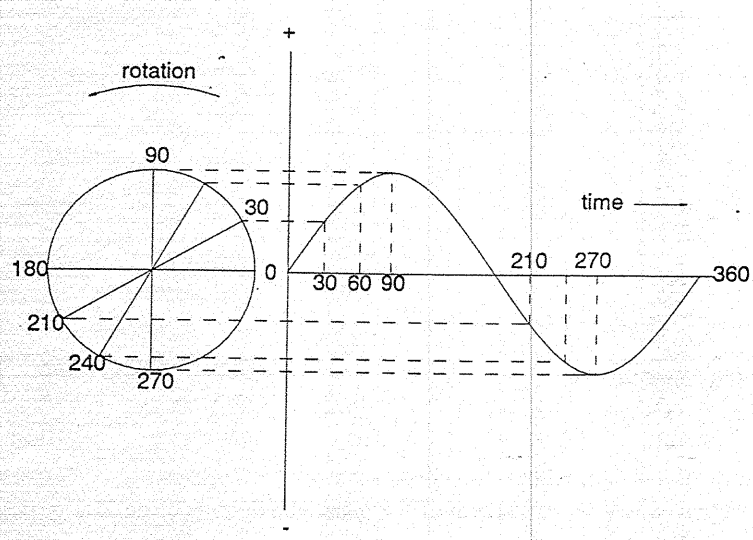


Figure 7 - *plotting a sine wave*

The mass will move 30 degrees (one "o'clock" position) every second; and it will take 360 degrees (or 12 seconds) to make one complete revolution. At time 0 we draw a line over to the coordinate axes and plot the position 0 along the x axis (since the x axis is time).

One second later the mass has moved 30 degrees, and is at the 2:00 position. As we draw a line over to the graph we can see that the line intersects the y axis in a positive direction. We move farther over on the time axis (since one second has passed) and plot the point.

If we do this for one complete revolution, we will draw a sine wave just like our pendulum did. Notice that the x axis in Figure 7, however, is labeled in degrees instead of seconds. The x axis still represents time, only now time is being measured in degrees! This shouldn't be hard to understand since, in our example, we said that the mass moved 30 degrees every second. It is useful in vibration analysis to measure time along the x axis in degrees. In Section 13 we will show how this degree measurement makes calculations much easier.

Baron Jean Baptiste Joseph Fourier (1768-1830) proposed the idea that the motion of a system in equilibrium with a complex waveform could be represented as the sum of many different sine waves of varying frequencies and amplitudes. So even though a vibration signature rarely looks as clean as a pure sine wave, a system's response to vibration can be explained using sine waves.

Look again at Figure 6. When the amplitude is at its maximum positive value, the pendulum is at its rightmost position. There the velocity is 0. As the pendulum moves back to the 0 line, its displacement is now 0 but the velocity is maximum.

If we put a plot of the velocity on the same graph as the displacement we see an interesting phenomenon: the velocity is 90 degrees ahead of the displacement. Look at the *y* axis of Figure 8. Here the velocity is at its maximum but the displacement is 0. Then look along the *x* axis 90 degrees later. Now displacement is maximum (it has "caught up" with where the velocity was), but velocity is 0. This relationship is repeated throughout the curve; the velocity will always lead the displacement by 90 degrees.

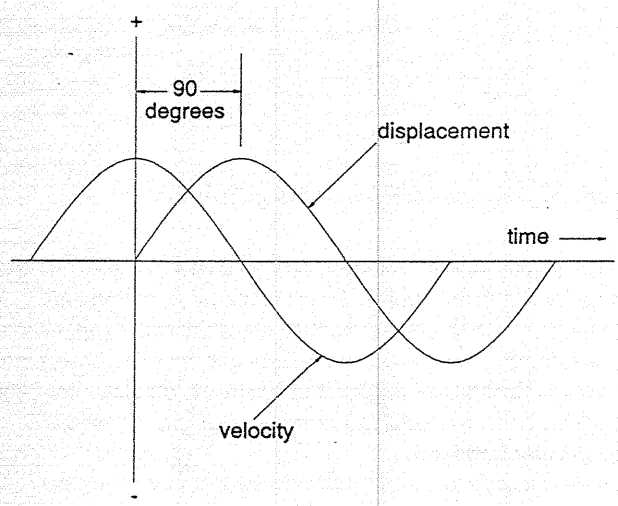


Figure 8 - *the relationship of velocity and displacement*

When the pendulum moves toward 0 from the leftmost position it increases in velocity, reaching maximum velocity when it passes through the 0 displacement line. A change in velocity in unit time is acceleration. The two curves in Figure 8 do indeed show the acceleration, since velocity increases as displacement approaches zero (and decreases as displacement increases).

To see this, pick a point along the *x* axis. Draw a line perpendicular to that point so that the line intersects both the displacement and velocity curves. The point of intersection on the curves represents the value of displacement and acceleration at that moment in time. Now move your point to the right to a later moment in time and find the points on the curves. You will see that if the displacement value increased, then the velocity value decreased. Likewise, if the displacement value decreased, then the velocity value increased.

When the pendulum passes through the 0 line it slows in velocity (that is, decelerates) as it moves toward the rightmost position. The acceleration is repeated as the pendulum returns to the 0 line from the rightmost position. The pendulum accelerates positively (toward the positive side of the 0 line) when it is on the negative side. Likewise the pendulum accelerates negatively (toward the negative side) when it is on the positive side. Looking back again at the pendulum, we see that from the leftmost position the pendulum accelerates positively toward the 0 point.

If we add a plot of acceleration to the displacement and velocity curves, we get the graph shown in Figure 9. The plot in Figure 9 shows that acceleration is directly opposite displacement, and that acceleration leads velocity by 90 degrees (and displacement by 180 degrees).

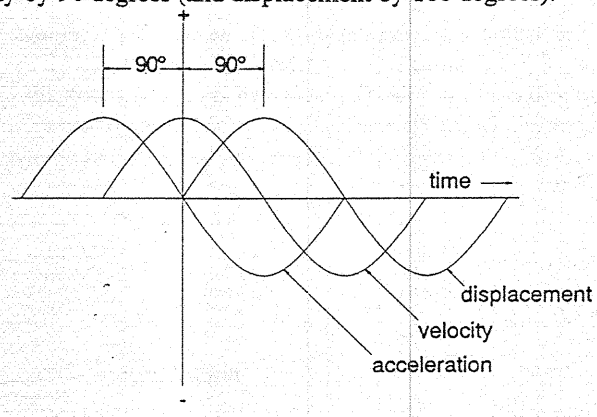


Figure 9 - *the relationship of acceleration, velocity, and displacement*

This relationship between displacement, velocity, and acceleration gives us a new term: **phase**, denoted by the Greek letter ϕ (pronounced "phi"). Phase is the time relationship between two related events, and is expressed in degrees. When two events occur at the same instant in time they are said to be "in phase." Events that do not occur at the same time are "out of phase," and their separation in time (measured in degrees) is their phase angle. For example, displacement and velocity are 90 degrees out of phase, or have a phase angle difference of 90 degrees. Phase gives us another term to describe vibratory motion: circular frequency.

Circular frequency (ω) - A measure of the amount of revolution in unit time. Circular frequency is written as the Greek letter ω (pronounced "omega"), and equals $2 * \pi * f$. Circular frequency is measured in radians/second in both the English and metric systems. A **radian** is an angular measurement that is often used instead of degrees. Figure 10 shows two circles of equal radius centered on the origin of a set of x,y axes. Angle A is made by drawing two radii from the center of the circle to the edge. The arc that is made by these two radii (labeled α) has a certain length. If we cut α out of the circle and hammered it flat, we could use a ruler to measure the length of the arc.

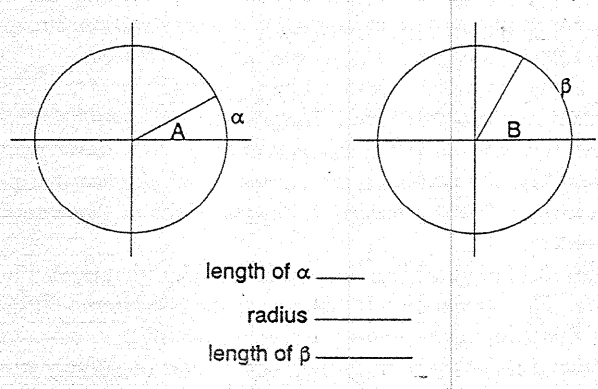


Figure 10 - *a radian*

Now look at angle B. The arc that is cut by these two radii (labeled β) is much longer than α. In fact, if we measured the length of β, we would find that it is the exact same length as the radius of the circle. This angle B is equal to one radian, and 1 radian equals approximately 57 degrees.

So if a circle is 360 degrees, how many radians is it? Remember that the circumference of a circle is equal to 2 *π * *r* (the radius of the circle). Since an angle of one radian cuts an arc equal in length to the radius, there are 2π of those arcs in a circle. Therefore there are 2π radians in a circle.

6.0 NATURAL FREQUENCY

If we pull the pendulum back and release it, it will swing back and forth at its natural frequency. The natural frequency f_n of the pendulum (in Hertz) can be found by the formula

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \qquad \text{(Formula 2-1)}$$

MEASURING AMPLITUDE

There are two other methods that may be used to express the amplitude of the wave: average and RMS.

The average of a wave (denoted as A) can be found by sampling the wave at several points and taking the average of these values. Since part of the wave is positive and part is negative, the positive and negative values cancel each other out when added. To avoid this, the waveform is rectified (made entirely positive) before the average is taken. For a pure sine wave, A is equal to .637 times the peak value.

The RMS (root mean squared) amplitude of a complex (i.e. distorted) wave is a much more accurate measure of the wave's energy content. The energy of a wave is equal to the area under the curve; the area under the positive part of the curve (above the 0 line) is positive and the area under the negative part of the curve is negative.

If we were to add up these two areas the answer would be 0, since half of the wave is positive and half is negative. In order to get a non-zero answer, we first square every value on the curve (hence the S in RMS). This squares the height of the wave on both sides of the 0 line, but also brings all the negative "humps" up to the positive side.

Now we can add the areas together since they are all positive. We then find the mean (average) of the curve over time by dividing by the period (hence the M in RMS). Finally, since all the areas are the square of those of the original wave (since we squared the wave), we need to take the square root of the mean (the R in RMS).

For pure sine waves, the RMS amplitude is equal to .707 times the peak value. More complicated waveforms will have an RMS different than .707 times the peak. Since the Peak amplitude is 1.414 times greater than the RMS amplitude (because RMS is 70.7% of the peak), it is very important to know what kind of amplitude values you are reading.

We can apply any of these amplitude measurements (Peak, Peak-to-Peak, Average, RMS) to displacement, velocity, and acceleration as well. Industry normally measures as follows:

amplitude - measured as Peak
displacement - measured as Peak-to-Peak
velocity - measured as Peak
acceleration - measured as RMS

where

f_n = the natural frequency (Hz)
g = acceleration due to gravity (ft./sec^2 or meters/sec^2)
l = length of the pendulum (feet or meters)

For example, what is the natural frequency of a pendulum one meter long? Remembering that the acceleration due to gravity is 9.8 m/sec^2, the quantity g/l is 9.8/1 or 9.8. The square root of 9.8 is roughly equal to 3.13. Now we multiply 3.13 by $1/2\pi$, which is roughly equal to .159, so the natural frequency is equal to .159 * 3.13 = .498 Hz (or nearly one half cycle per second. Low, but there nonetheless).

AMPLITUDE, VELOCITY, AND ACCELERATION

There is a mathematical relationship between amplitude, velocity, and acceleration. Therefore if one value is known the other values can be found. To explore this relationship, let's look again at the sine wave plot of vibratory motion. The instantaneous amplitude x is equal to the amplitude at any instant in time. The maximum amplitude X is equal to the maximum distance from the 0 line. The instantaneous amplitude x is related to the maximum amplitude X by:

$$x = X \sin(\omega t)$$

Velocity is the change in position (amplitude) over unit time, or

$$v = \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Therefore $v = \omega X \cos(\omega t) = \omega x \sin\left(\omega t + \frac{\pi}{2}\right)$

The cosine indicates that velocity is 90 degrees out of phase from amplitude. Acceleration is then equal to dv/dt, or

$$a = -\omega^2 X \sin(\omega t) = \omega^2 X \sin(\omega t + \pi)$$

the negative sine indicates that acceleration is 180 degrees out of phase from amplitude.

The sine, cosine, and -sine terms all indicate that these are the instantaneous values for amplitude, velocity, and acceleration. If we remove the instantaneous components we are left with a simple relationship between amplitude, velocity, and acceleration:

amplitude = x
velocity = ωx
acceleration = $\omega^2 x$

Now let's use this relationship. Suppose a vibrating system has an amplitude of .010 (10 mils) when operating at 100 Hz. What is the velocity and acceleration? Since $\omega = 2\pi * f$,

$v = 2 * \pi * 100 * .01$
$= 6.28$ in/sec

$a = (2 * \pi * 100)^2 * .01$
$= 3{,}945.84$ in/sec^2

Acceleration is usually measured in g's. One g is equal to 386.4 in/sec^2, so

$a = 3{,}945.84/386.4$
$= 10.21$ g's

If you want to express the natural frequency in radians/second, the formula is:

$$\omega_n = \sqrt{\frac{g}{l}} \qquad \text{(Formula 2-2)}$$

where

ω_n = natural frequency (radians/sec)
g = acceleration due to gravity (ft./sec^2 or meters/sec^2)
l = length of the pendulum (feet or meters)

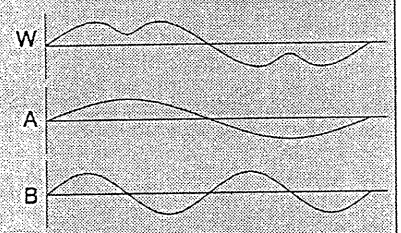
Physics textbooks usually express natural frequency in radians/second, while most technicians in industry prefer to express the natural frequency in cycles per second (Hz).

In these two formulas, the only thing that determines the frequency of the pendulum is its length. The quantity $1/2\pi$ never changes; neither does g, the acceleration due to gravity. So the only quantity in the formula that can vary from pendulum to pendulum is length.

HARMONIC DISTORTION

According to Fourier, if a complex waveform has no discontinuities (i.e. random, unpredictable events), then it can be represented as the sum of individual sine and cosine waves of varying frequencies and amplitudes. The process of breaking down the complex waveform into simple, related sine waves is called a Fourier Transformation.

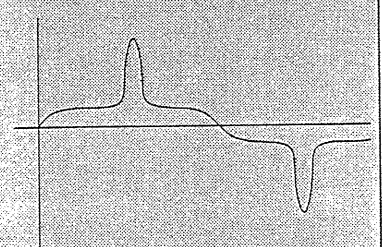
The simple component waves that result from a Fourier Transformation are all related to each other. For example, waveform *W* shown in (a) is composed of the two pure sine waves *A* and *B*. The frequency of wave *B* is twice that of wave *A*, so *B* is said to be a harmonic of *A*. We could just as easily have looked at wave *B* first and said that wave *A* is half the frequency of *B*. In this case *A* would be called a sub-harmonic of *B*.



(a)

Pure sine waves become complex waveforms through harmonic distortion; the mixing of several related sine waves (harmonics). The amplitude measurements of Peak, Peak-to-Peak, and Average can all become error-prone as a result of harmonic distortion.

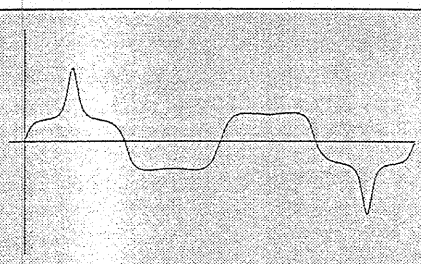
For example, waveform (b) is characterized by fast rise, short duration spikes. What would you call the peak of this wave? The top of the spike? The top of the broad plateaus that surround the bottom of the spike? Both of these definitions of Peak make sense, but they are very different.


(b)

This difference can result in two different average (A) values, which is found by multiplying .637 times the peak. Which peak do you use? Given the separation of these two possible peaks, your average value will differ drastically from one to the other.

Consider a swing hanging from a rigid support. If you want the swing to swing quickly, you make the ropes very short, say only 3 feet long. If you want it to swing very slowly, you could hang the swing into the Grand Canyon and make the ropes 5000 feet long. In both cases the weight of the person sitting on the swing is not important. Only the length of the rope determines the frequency.

Industry is often concerned with Peak-to-Peak measurements, as these give an indication of how much a system is moving. Waveform (c) shows that in one cycle the system moved quite a bit in the positive direction and very little in the negative direction. In the next cycle the spike is in the negative direction, with very little movement in the positive direction.



(c)

How would you determine Peak-to-Peak amplitude on this waveform? If you look at only one cycle the Peak-to-Peak amplitude is much smaller than if you looked at the total distance between the maximum positive movement and the maximum negative movement. But Peak-to-Peak amplitude, by definition, is the total movement within one cycle. You will sometimes hear people refer to the *true peak*, which is the distance between the maximum positive and maximum negative peaks within the sample.

Harmonic distortion has a big effect on amplitude measurements. How do you decide what is a Peak? Do you measure Peak-to-Peak within one cycle or within the entire sample? How accurate is your Average? RMS is the only amplitude measurement that is unaffected by harmonic distortion. You might be wondering how this can be, since we said earlier that RMS is equal to .707 times the peak. This is only true for pure sine waves; for complex waveforms the RMS will be either lower or higher than .707 times peak.

The average of a complex waveform depends on the phase relationships between the various harmonics. These phase relationships cancel each other out in places, and magnify themselves in others. This interference affects the average value of the resultant waveform.

The RMS value is unaffected by phase relationships between harmonics; RMS is only concerned with the area under the curve as a result of the harmonics. The process of calculating the RMS is still the most accurate method of determining the energy of vibration, regardless of whether the vibration is purely sinusoidal or complex.

The **crest factor** of a wave is its peak value divided by its RMS value. Since, for a pure sine wave, RMS = .707 * peak,

$$\text{peak/RMS} = 1/.707 = 1.414$$

Crest factor is useful if you do not have a time waveform of the vibration of your system. Some vibration analyzers do not give you a time waveform display; instead, they only give you a crest factor reading. If your vibration instruments read out in crest factor, you know that a reading of 1.414 means that your vibration waveform is a pure sine wave. A reading much higher than 1.414 means that the waveform has fast rise/short duration peaks, where the peak is very high but the RMS value is low due to the small area under the curve. Readings lower than 1.414 mean that your waveform approaches a square wave, where the peak is low but the RMS is rather high due to the large area under the curve.

7.0 DAMPING

Notice that there are no minus signs in Formulas 2-1 and 2-2. We have assumed that this pendulum has no sources of damping (friction at the pivot, air resistance, and so on). If nothing is being subtracted, then there is no energy loss and the pendulum will swing forever. We know from experience, however, that nothing goes forever. We can state one of the five rules of system response:

1. ALL SYSTEMS HAVE DAMPING

UNITS OF MEASUREMENT

When solving formulas, you must always remember to use the correct units! Otherwise, if the units don't match, your answers will be incorrect.

Formula 2-1 shows that the natural undamped frequency of a pendulum is found by

$$\frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

The quantity $1/2\pi$ has no units. The two variables g and l, however, have units. The acceleration due to gravity g may be expressed in either m/sec^2 (meters/second^2) or ft/sec^2 (feet/second^2):

g = 9.8 m/sec^2
= 32.2 ft/sec^2

Note that both these figures are approximate, as the force of gravity varies with your altitude and position on the Earth. These two values are generally accepted by the scientific community as accurate and are used in most calculations. If we were calculating satellite orbits we would want more accuracy, but for our purposes here these values are just fine!

It does not matter which of these values you use, as long as you are consistent with the other terms in the formula. Since the units of g can contain either meters or feet, the length l must be measured in the same units:

If g is measured in m/sec^2, measure l in meters
If g is measured in ft/sec^2, measure l in feet

This way the length units will cancel each other out and you will be left with 1/sec^2 under the radical. The square root of 1/sec^2 is 1/sec, leaving you with an undamped natural frequency in cycles per second.

Our example calculation of the natural frequency of a pendulum one meter long used the value 9.8 m/sec^2 for g since the length of the pendulum was in meters. The natural frequency of this pendulum was .498 Hz.

What if we performed the same calculation but chose to express g in ft/sec^2 instead? First, we would have to convert the length of the pendulum into feet. One meter equals 3.281 feet, so g/l equals 32.2/3.281 or 9.814. This is within .014 of the 9.8 we had in our previous example, the slight difference due to g being slightly more than 32.2 ft/sec^2 and to the inevitable inaccuracy when converting from feet to meters.

Taking the square root and multiplying by $1/2\pi$ gives us

.159 * 3.12 = .496 Hz

Notice that our undamped natural frequency differs from the previous example by only .002, or .2%

In order to make these formulas more true to life, we must include some term that represents energy loss from damping. Damping is represented mathematically by the letter *c*.

Viscous damping is defined as force/velocity, and is measured in English units of pound-seconds per inch (lb.-sec./in.) Metric units are gram-seconds per centimeter (gm.-sec./cm.). To add damping to the formula, we change Formula 2-1 to become

(Formula 2-3)

$$f_c = \frac{1}{2\pi}\sqrt{\frac{g}{l} - \left(\frac{c}{2m}\right)^2}$$

where

f_c = damped natural frequency (Hz)
g = acceleration due to gravity (cm/sec^2)
l = length of the pendulum (cm)
c = damping (gm-sec/cm)
m = the mass of the pendulum ($\frac{gm\text{-}sec^2}{cm}$)

Originally, when we assumed that there was no damping in the pendulum, we didn't care about the mass; the only thing that determined the pendulum's frequency was the length. Now, when calculating the damped natural frequency, the mass is important.

The minus sign in Formula 2-3 allows us to look at the effect of damping on the natural frequency of the system. If the quantity g/l is greater than the quantity $(c/2m)^2$, then their difference will be some positive number and the damped natural frequency will be calculable. For example, if g/l equals 9.8 and $(c/2m)^2$ equals 2.25, then 9.8 - 2.25 = 7.55.

Continuing on, the square root of 7.55 roughly equals 2.75, and finally 2.75 * $1/2\pi$ = .437 Hz, or a damped natural frequency of .437 cycles per second. Notice that this is indeed lower than the previous undamped frequency of .498 cycles per second.

When g/l is greater than $(c/2m)^2$, their difference will always be a positive number and the square root of their difference will be greater than 0. Systems with a damped resonant frequency greater than 0 are **underdamped**. Underdamped systems are free to vibrate, and, when disturbed, will vibrate through several cycles before coming to rest. Figure 11 shows a typical amplitude vs. time plot for an underdamped system.

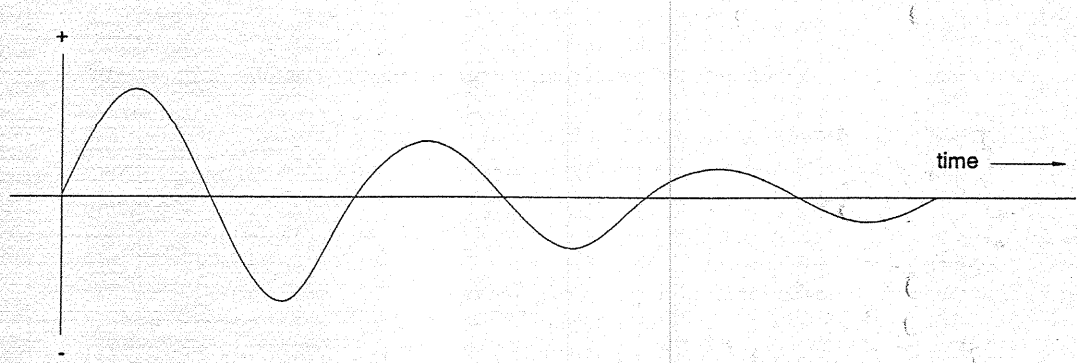


Figure 11 - *an underdamped system*

As g/l and $(c/2m)^2$ get closer to each other, their difference becomes much smaller. If the quantity g/l equals the quantity $(c/2m)^2$, then the result is 0 when these two are subtracted. The square root of 0 is 0, and 0 times $1/2\pi$ equals 0.

A system that has a damped resonant frequency equal to 0 is **critically damped**. A critically damped system will move through one cycle but it will not vibrate. Consider a pendulum hanging in a swimming pool. If the pool is empty the pendulum is underdamped and will vibrate freely when pushed. If we fill the pool with water we now have a system that is near critically damped. Given the same push the pendulum moves slightly and then stops. It may move through at most one cycle, but the vibratory amplitude will be low. It would take a lot of energy to keep this system moving.

Figure 12 shows a typical amplitude vs. time plot for a critically damped system. Notice that the amplitude begins to rise and fall like the periodic displacement curve of an underdamped system, but then stops after only one cycle.

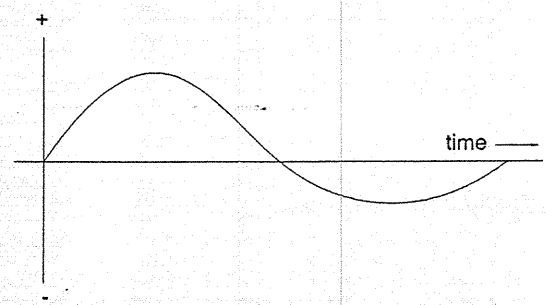


Figure 12 - *a critically damped system*

If g/l is less than $(c/2m)^2$, the result is a negative number and the square root of a negative number is imaginary. Systems whose damped resonant frequency is imaginary are **overdamped**. Overdamped systems may also move, but their motion is non-oscillatory. Figure 13 shows a typical amplitude vs. time plot for an overdamped system. Notice that the motion doesn't even begin to vibrate; once disturbed the system drops very quickly to 0.

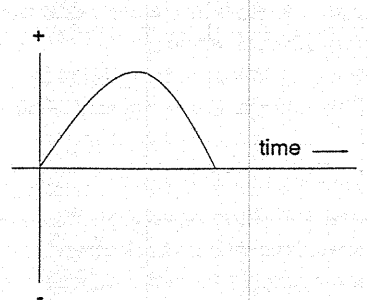


Figure 13 - *an overdamped system*

Our pendulum is a good example of an underdamped system. When disturbed by an initial force, it will vibrate several times before coming to rest. With few exceptions, most mechanical systems in industry are also underdamped; when forced, they vibrate with little damping.

Some auto manufacturers design suspension systems that are critically damped. When the car hits a bump on the road, the shock absorber vibrates once and comes to rest. The huge scales at truck weigh stations are overdamped. When a truck pulls onto the scale the scale comes to rest very quickly. If the scale were free to vibrate it would take much longer to weigh the truck.

Now let's use our spring-loaded pen to make a plot of a damped pendulum's movement. Just as before, we pull the paper out at a constant rate while the pendulum vibrates. But now, since the pendulum has damping, it will eventually stop vibrating and the plot will look like the one shown in Figure 14.

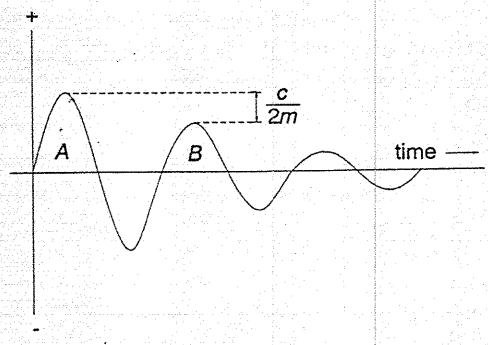


Figure 14 - *plotting a damped pendulum's movement*

Notice how the damping causes the amplitude of each cycle to be lower and lower, until the curve finally meets the 0 line and the pendulum stops vibrating.

Since the amplitude of each cycle decreases, the area under the curve decreases as well. The area under the positive portion of the first cycle is marked A in the figure; the same area in the second cycle is marked B. If we were to calculate the area of A and B and subtract them, the difference between them would be equal to $c/2m$. This continues for each remaining cycle; the area under the curve is decreased by $c/2m$ again and again until there is nothing left.

Now you can see how the term $c/2m$ is involved in damping vibration. Area A is the energy that was put into the pendulum to get it moving, and energy B is what we got back; the rest of the energy was absorbed by the damping. To keep the pendulum vibrating at the same amplitude, energy equal to $c/2m$ must be put back into the system for each cycle.

If c is very low, then the quantity $c/2m$ is low and damping is low. Although most systems have very little damping, some amount of damping is always present. Our second rule of system response is then:

2. MOST SYSTEMS HAVE SMALL AMOUNTS OF DAMPING

In most cases, systems are designed with little or no regard to damping. The only exceptions to these are systems such as vehicle suspensions where shock absorbers are used to dampen vibrations. But these are exceptions; systems are designed to move as freely and unimpeded as possible, so consequently they have small amounts of damping. This is an important consideration, since small amounts of damping do very little to dampen unwanted vibration.

Since damping is rarely designed into a system, the major sources of damping in industry are lubricants and the process. Lubricants are usually rather viscous liquids that dampen vibration. Processes like pumping air or water also provide damping.

When a system is vibrating at its natural frequency, and the energy loss from damping is small, it's easy for the system to vibrate. Therefore, when a system is vibrating close to or at its natural frequency, the energy required to keep it moving is very small.

If we want to keep a damped pendulum vibrating at its natural frequency, we must put energy into it (that is, give it another little push) each cycle. However, this additional energy (equal to $c/2m$) is much lower than the energy that we put in to get the pendulum moving in the first place. Therefore we can state rule number three as:

3. ENERGY REQUIRED TO KEEP A SYSTEM VIBRATING IS LOW

This is very obvious when you think about pushing a child on a swing. The first push is the hardest, but from then on you only need to push lightly to keep the child moving.

8.0 FREE VS. FORCED VIBRATION

You will often hear people in the vibration discipline talk about free and forced vibration. In the systems we've talked about up to now, an initial force was put into the system to start it vibrating (the pendulum was pulled back, the child on the swing was pushed). Once the system began to vibrate, it was left alone. No additional energy was added, and the system eventually stopped vibrating as a result of damping. This type of vibration is free vibration: no energy is added after the initial force that started the system vibrating.

Forced vibration is maintained from cycle to cycle by a forcing function. Some part of the process (a motor, a turbine, and so on.) forces the system to vibrate, and maintains the energy so that the amplitude remains the same. Mathematically the resonant frequency f_r of a system being forced to vibrate can be expressed as

$$f_r = \frac{1}{2\pi}\sqrt{\frac{g}{l} - \left(\frac{c_1}{2m_1}\right)^2 - \left[\left(\frac{c_2}{2m_2}\right)^2 - \cdots - \left(\frac{c_n}{2m_n}\right)^2\right]} \qquad \text{(Formula 2-4)}$$

where

f_r = resonant frequency of forced system (Hz)
g = acceleration due to gravity (cm/sec^2)
l = length of the pendulum (cm)
c_1 = damping at the first source (gm-sec/cm)
m_1 = the first mass (gm-sec^2)
c_2 = damping at the second source (gm-sec/cm)
m_2 = the second mass (gm-sec^2)
c_n = damping at the nth source (gm-sec/cm)
m_n = the nth mass (gm-sec^2)

Notice the additional terms inside the braces. This formula is a simplified expression of the damped resonant frequency when there are multiple masses and multiple sources of damping.

To illustrate the use of Formula 2-4, let's modify our pendulum a little bit. First we'll add an arm to one side of the pendulum, and connect the arm to a slot on a wheel. Now we'll take a motor and connect the motor shaft to the wheel. The slot allows us to vary the amplitude of vibration: with the arm connected to the outermost portion of the wheel we get maximum amplitude. If we move the arm down the slot closer to the center of the wheel the amplitude will be lower.

This system now has several masses and several sources of damping:

- the damping in the pendulum itself. We will call this damping c_1 and the mass m_1.
- damping due to friction in the arm connecting the pivot to the wheel. This is damping c_2 and the arm is mass m_2.
- damping in the central pivot on the wheel. This is damping c_3 and the wheel is mass m_3.
- damping within the motor. This is damping c_4 and mass m_4.

Putting all these into Formula 2-4, we now have

$$f_r = \frac{1}{2\pi}\sqrt{\frac{g}{l} - \left(\frac{c_1}{2m_1}\right)^2 - \left(\frac{c_2}{2m_2}\right)^2 - \left(\frac{c_3}{2m_3}\right)^2 - \left(\frac{c_4}{2m_4}\right)^2}$$

This is a very complicated equation, and would take some time to determine the damping at each source. Normally in industry it is not necessary to determine all sources of damping and calculate such a complex equation. Since damping is small, the difference between the simpler Formula 2-3 and the more complex Formula 2-4 would be small.

9.0 FORCED VIBRATIONS

A pendulum is not the best representation of vibrating systems found in industry. Mechanical systems can be better illustrated as a "spring-mass" system suspended from (or mounted on) a fixed point as shown in Figure 15.

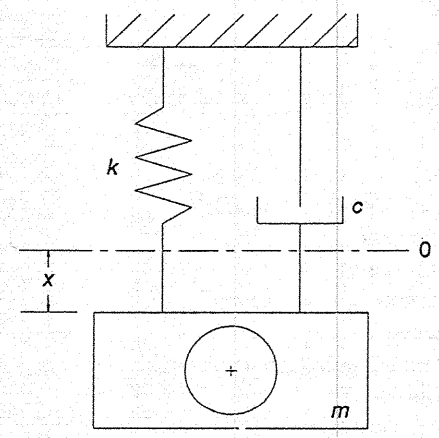


Figure 15 - *a spring-mass system*

A mass m is suspended from a fixed point by a spring and a damping mechanism. The spring has a spring constant k, and the damping mechanism has a constant c. We will call the "at rest" line 0, as shown in the figure. Movement above the 0 line will be positive, and movement below the 0 line will be negative.

If the mass were pulled down and released, you would have free vibration at the system's damped natural frequency. The mass would bob up and down until the damping brought it to a halt. The undamped natural frequency (in Hertz) of a spring mass system is found by the formula

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \text{(Formula 2-5)}$$

where

f_n = undamped natural frequency (Hz)
k = the spring constant (gm/cm)
m = the mass (gm-sec^2/cm)

Notice that this formula is very similar to Formula 2-1. Gravity provided the spring for the pendulum, but now we have a spring of constant k instead. But this system has damping; all systems have damping. Including damping in the formula is exactly like the damped frequency shown in Formula 2-3:

$$f_c = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2} \qquad \text{(Formula 2-6)}$$

where

f_c = damped natural frequency (Hz)
k = the spring constant (gm/cm)
m = the mass (gm-sec^2/cm)
c = damping (gm-sec/cm)

where the damping component is equal to $\left(\frac{c}{2m}\right)^2$

Finally, the forced resonant frequency of the spring mass system is found by

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \left(\frac{c_1}{2m_1}\right)^2 - \left[\left(\frac{c_2}{2m_2}\right)^2 - \cdots - \left(\frac{c_n}{2m_n}\right)^2\right]} \qquad \text{(Formula 2-7)}$$

where

f_r = forced resonant frequency (Hz)
k = the spring constant (gm/cm)
m = the mass (gm-sec^2/cm)
c_1 = damping at the first source (gm-sec/cm)
m_1 = the first mass (gm-sec^2)
c_2 = damping at the second source (gm-sec/cm)
m_2 = the second mass (gm-sec^2)
c_n = damping at the nth source (gm-sec/cm)
m_n = the nth mass (gm-sec^2)

The information inside the braces contains other damping and other masses that are part of the forcing function.

Now imagine that there is a motor inside the mass (or that the mass is the motor), and that the motor has some imbalance. As the motor turns, the imbalance creates a force that varies with time; at any time the force will be pointing off at some angle, like the second hand on a watch sweeping around 360 degrees.

The imbalance in the rotor will provide a forcing function when the motor is turning. This forcing function will cause the mass to move up and down at the frequency of the forcing function, not at the natural frequency.

10.0 PLOTTING FORCED VIBRATION

Since our spring-mass system will be forced to vibrate by the motor, let's make a plot that shows how the system will respond to a wide range of motor frequencies.

We start by attaching a variable speed control to the motor. Now we construct a graph that has motor frequency along the *x* axis and mass amplitude along the *y* axis. When we run the motor up to its operating speed, starting at 0 Hz and slowly increasing the frequency, we measure the amplitude and plot it on the graph. When we are done the graph looks something like Figure 16.

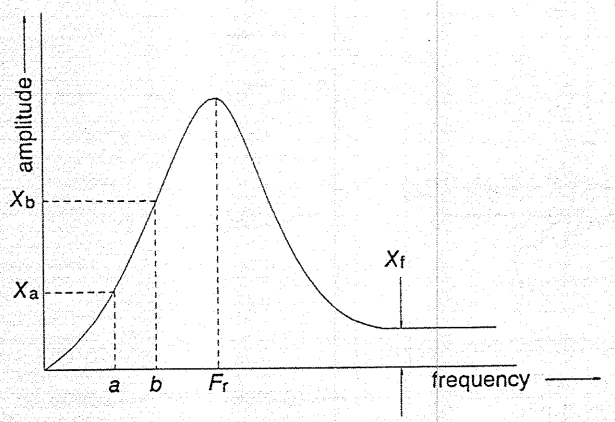


Figure 16 - *amplitude vs. frequency response curve*

This graph represents a system response for a constant input of energy. In the case of our spring-mass system, the energy input comes from imbalance in the motor. We will also assume for the moment that the input energy is constant over the frequency range. For a given amount of energy, then, we can obtain a wide range of amplitude simply by varying the frequency. For example, running the motor at frequency *a* as shown in the figure gives an amplitude of X_a. A greater amount of movement can be obtained (that is, a higher amplitude) at the same energy input by running the motor at a higher frequency. In this case, running the motor at frequency *b* gives an amplitude of X_b.

The dotted line down the middle of the graph, cutting right through the peak of the curve, is F_r, the **resonant frequency**. At this frequency the forcing function causes the greatest amount of movement. Operating a system at its resonant frequency always results in the system's maximum amplitude.

Finally, notice that the curve starts at 0, in the origin of the graph. This makes sense, because at this point the motor is off and there is no vibration. As the frequency increases, the amplitude

increases. When the frequency reaches F_r, the forced natural damped frequency, the amplitude is at its maximum.

The amplitude then begins to drop as the frequency increases above F_r. But notice that the amplitude never returns to 0. Rather the curve flattens out into a straight line. This value represents the amplitude with no amplification due to resonance, and is called X_f or the **free space amplitude**.

10.1 Amplification

We have stated several times that the frequency response curve in Figure 16 is for a constant input of energy. If the energy input is constant, then why does the amplitude change so much? The answer is that, at or near F_r, the system actually amplifies its response to the input energy from the forcing function.

If we draw a horizontal line over from the free space amplitude toward the *y* axis, the line intersects the curve in two places as shown in Figure 17.

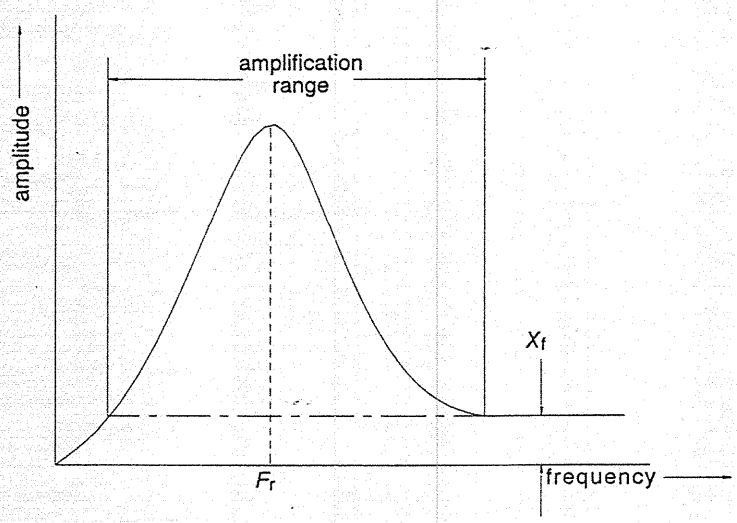


Figure 17 - *the amplification range*

These two points form the boundaries of the system's **amplification range** (also called the **critical speed range**). In the amplification range, the vibratory motion due to the forcing function is amplified by the system.

Below the amplification range, the amplitude varies with frequency until it reaches the free space amplitude. If the frequency continues to increase, it enters the amplification range. Now the amplitude is amplified by the system to a value above the free space amplitude. The maximum amplification occurs at F_r, near the middle of the amplification range.

As the frequency increases through the amplification range, the amplitude drops below that at F_r. The amplitude continues to drop until it reaches the free space amplitude, at the end of the amplification range. Now the curve is flat because no amplification occurs outside the amplification range unless there is another degree of freedom in the system (see Section 11.0).

If a forcing function frequency is inside the amplification range of a system, the system will amplify the energy of the forcing function. If the frequency is close to either boundary of the range the

amplification will be small. If the frequency is close to F_r (or at F_r), then the amplification will be rather large, with potentially disastrous results.

This is how opera singers are able to break a glass by singing a high note. The frequency of the note is very near the resonance of the glass, causing the glass to vibrate until it shatters. Army troops marching over bridges are usually ordered to break step, so that the rhythmic pounding of their feet does not drive the bridge into the resonant amplification range. At least two bridges are known to have collapsed in this way during the mid-nineteenth century: one in Manchester, England and one in Anjou, France.

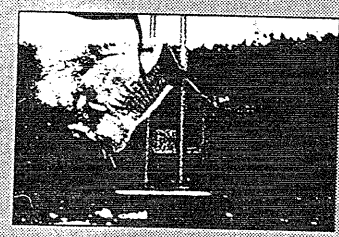
THE TACOMA NARROWS BRIDGE - RESONANT AMPLIFICATION

The Tacoma Narrows bridge in Washington opened for traffic on July 1, 1940. Compared to other suspension bridges at that time, the Tacoma Narrows bridge was unusually long and narrow. The original design had called for trusses to stiffen the span. However, funds were not available and less expensive stiffening was used. Unfortunately this stiffening was inadequate.

Suspension bridges have aerodynamic qualities. Wind blowing over and under the deck creates lift, causing the bridge to move up and down. Relatively continuous wind along Puget Sound would lift the bridge; as the bridge moved up, the angle of attack would change, decreasing lift until gravity pulled the bridge back down. Even though the excitation force was not periodic, the bridge moved up and down at its natural frequency. And since the bridge was moving at its natural frequency, the resonant frequency of the structure amplified the movement.

In the four months of the bridge's life, amplitudes of 5 feet were observed along the bridge's center span. Motorists crossing the bridge reported seeing the car ahead disappear from view and then reappear due to the extreme vibratory amplitude.

On November 7, 1940, the winds were blowing at 40-45 mph. Unusually large amplitudes were reported and the bridge was closed to traffic. Around 10:00 AM the collar around the north cable (from which the bridge was suspended) came loose. The span began to vibrate around its center line up to about 35 degrees from the horizontal. The span broke up and collapsed shortly after 11:00 AM.



The free space amplitude X_f is caused by the imbalanced forcing function of the motor. The amount of this force is found by the formula

$$F_u = \frac{\omega^2 Ub}{g} \qquad \text{(Formula 2-8)}$$

where

F_u = force due to imbalance Ub
ω^2 = the square of the circular frequency of the motor (radians/sec)
Ub = the amount of imbalance (gm-cm)
g = acceleration due to gravity (cm/sec^2)

This is a force, and this force must be equal to mass times acceleration $\left(F = \frac{\omega^2 Ub}{g} = ma\right)$. If a force is applied to a mass, the mass will move. How much the mass moved, how much its velocity changed, is acceleration. Therefore the product of the mass and its acceleration is a measure of the force that was applied.

If we substitute for mass and acceleration, we can obtain a formula that is very useful in vibration analysis. Mass is defined as w/g, the mass's weight divided by gravity. Remember, weight is only a result of gravity. Acceleration is defined as $\omega^2 x$ (the square of the circular frequency multiplied by the amplitude). We can use the free space amplitude X_f for the amplitude.

NORMALIZING THE RESPONSE CURVE

We have said that the system response curve shown in Figure 16 is for constant input energy. Formula 2-8, however, shows that the imbalance force is not constant if ω^2 is increasing.

Formula 2-8 contains two constants: the rotor imbalance Ub and g, the acceleration due to gravity. The only variable in Formula 2-8 is ω^2, the circular frequency. If we slowly increase the frequency of the rotor (as we did in section 10.0), then the quantity ω^2 increases. And as ω^2 increases, the imbalance force increases proportionally.

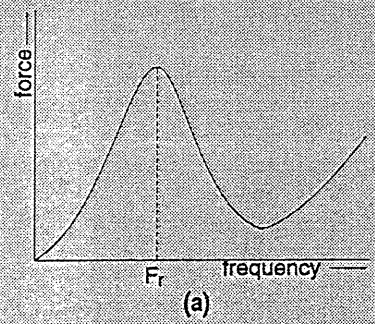
So how can this be constant input energy when the imbalance force increases as the frequency increases? What needs to be done (and what we have done) is to normalize the response curve. Normalize means "make constant," so somehow we must devise a way to make the energy constant across the curve.

We do this by dividing frequency out of the curve. We saw earlier that acceleration is the second derivative of displacement (position). Therefore double integration of acceleration will result in displacement. Double integration of acceleration is a division by ω^2, which removes the frequency component.

If we want to measure the response of a spring-mass system, we first need to excite it to vibrate. A small variable-speed motor that is deliberately made out of balance (by mounting a small weight, such as a bolt, off one side of the shaft) is a good exciter. We then use an accelerometer to measure the force of the imbalance. The response plot is shown in (a), where the x axis is frequency and the y axis is force (acceleration).

If we perform double integration on this response curve, we divide every value along the curve by ω^2. This results in displacement and the response curve we saw in Figure 16.

Once the frequency component has been removed from Formula 2-8 by normalizing, the imbalance force becomes Ub/g and the plot is now for constant input energy. Our normalized response curve still has that peak due to resonant amplification of the forcing function energy, but the free space amplitude beyond the amplification range is now constant.



Now that we have expressed the imbalance force, and have defined the mass and acceleration whose product equals that force, we can equate these two quantities by the formula F=ma (since $F = \frac{\omega^2 Ub}{g} = ma$). Note that they have to be equal since the system is in equilibrium.

$$\frac{\omega^2 Ub}{g} = \frac{w\,\omega^2 X_f}{g}$$

If we divide through by ω^2 and multiply through by gravity, we are left with

$$Ub = w * X_f \qquad \text{(Formula 2-9)}$$

where

Ub = the amount of imbalance (gm-cm)
w = the weight (gm)
X_f = the free space amplitude (cm)

This equation says that the imbalance is equal to the weight of the vibrating mass times the free space amplitude. Note that this equation has no frequency component. In the free space area of the response curve, when no amplification due to resonance is present, the vibration amplitude does not change with frequency.

A rotor will rotate about its mass center (its center of mass) when the rotational frequency is in the free space area of the curve (that is, above the amplification range). If the mass center of the rotor does not coincide with the bearing center, the vibration amplitude is simply equal to the mass of the

CALCULATING THE AMPLIFICATION FACTOR

The amplification factor of a system can be calculated by adding a known unbalance to a known rotor weight and then using Formula 2-10 to find X_f. For example, let's determine the amplification factor of a 20,000 ounce rotor. The rotor is first run up to speed and an "as is" imbalance reading is taken (we'll call this reading A_1). We then add an unbalance of 20 ounce-inches to the rotor. Using Formula 2-10 we find that the expected free space amplitude X_f will equal

X_f = 20/20,000
= .001 inch (1 mil)

Now we run the rotor up to operating frequency and take a reading A_2 of the actual vibration displacement caused by the addition of this weight. Reading A_2 is the resultant vector between the unbalance vector and the trial weight vector.

Now in order to see exactly what effect the trial weight had on the system, we need to subtract vector A_1 from vector A_2 (section 13.0 explains resultant vectors and vector subtraction). This gives us a vector T that we can use to determine the amplification factor. Suppose that the magnitude of T is 6 mils. Since this is displacement, the amplitude X_m is 3 mils. By using Formula 2-10 we expected to only get 1 mil, but in reality we got 3 mils. Therefore the amplification factor Af is found by

$Af = X_m/X_f$
= 3/1
= 3

rotor times the distance between centers. This is true regardless of frequency as long as the frequency remains in the free space area of the curve.

To understand this, consider a 1,000 pound rotor that is moved off-center by .001 inch. If the rotor is rotating in the free space area of the curve then the imbalance that is present is equal to .001 inch times 1,000 pounds, or 1 pound-inch of imbalance. And as long as the rotor operates in the free space area, the imbalance will remain at 1 pound-inch.

We can manipulate this formula to solve for any one of the three variables if we know the other two. For example, the free space amplitude can be calculated by manipulating Formula 2-9 to read

$$X_f = \frac{Ub}{w} \qquad \text{(Formula 2-10)}$$

where

X_f = the free space amplitude (cm)
Ub = the amount of imbalance (gm-cm)
w = the weight (gm)

Formula 2-9 is very useful. With it you can

1. Estimate initial imbalance. If you know the weight of the system and the amount it is moving, then you can determine the imbalance by multiplying the two values.
2. Calculate trial weights for balance. This is explained in greater detail in the chapter on Field Balancing.
3. Evaluate the system. The formula lets you quickly determine if a vibration is due to imbalance or to resonant amplification. For example, a 2,000 horsepower motor generator is vibrating 47 mils (.047 inches). This is double amplitude, the total amount of movement. The natural inclination would be to suspect rotor imbalance, but Formula 2-9 will quickly tell you otherwise. Manufacturer's records reveal that the rotor weighs 16,000 lbs. The two bearing pedestals at either end of the motor generator weigh 1,200 pounds each, or a total of 2,400 pounds. The total weight of the system, then, is 18,400 pounds. To determine the imbalance, we multiply this weight by the amplitude (we must divide .047 in half first, since .047 is the double displacement):

Ub = 18,400 * .024
= 441.6 pound inches

This means that, if the cause of vibration were imbalance, an extra 441.6 pounds at 1 inch must be inside the rotor! This is out of the question, since no reasonable machining error could result in that much imbalance. More likely, the cause of vibration is a resonant bearing pedestal. This was a case encountered by the authors, and further investigation revealed that many of the welds on the bearing pedestals were cracked. When the welds were repaired the amplitude dropped to less than 1 mil.

11.0 RESONANCE AND DEGREES OF FREEDOM

Our spring-mass system is a system with only one degree of freedom; the mass can move only up and down. Systems with multiple degrees of freedom have a resonant frequency for each degree of freedom. If an amplitude vs. frequency plot were made for a multiple degree of freedom system, there would be one resonant peak for each degree of freedom.

Generally in industry we are only concerned with the lower degrees of freedom (for example, the first through the third), since these lower degrees of freedom are usually close in frequency to forcing functions that exist in operating machinery.

12.0 SYSTEM OPERATING FREQUENCY VS. RESONANT FREQUENCY

Understanding resonance is important, since the majority of systems run close to F_r, their forced resonant frequency. This occurs for three reasons:

1. Competition
2. Wear
3. Customer demand

12.1 Competition

Competition between manufacturers plays a major part in the operating frequency of systems. For example, a manufacturer could decide to design and build a fan that will last forever and is guaranteed not to vibrate. This fan would be made with a high-carbon steel shaft 6 feet in diameter. Each blade would be made of armor plate steel and weigh 4 tons. The fan would operate well below F_r, weigh 800 tons, and cost $750,000.

A fan this massive might seem ridiculous, and indeed it is. However, such a fan has several advantages: the shaft would not bend, it wouldn't vibrate (if properly balanced), and ash and rust accumulation on the 4-ton blades would do very little to cause imbalance.

But these advantages are outweighed by its size and cost. Another fan manufacturer could build a much smaller fan that weighs only 4 tons total, occupies much less space, and costs only $22,000. By removing so much stiffness, and therefore causing the fan to operate closer to F_r, this manufacturer has drastically reduced the cost of the fan and made it more competitive.

12.2 Wear

Wear causes a system that operated within tolerance when new to operate close to F_r. Figure 18 shows the response plot for a new rotor. The operating frequency F_o is well below F_r, therefore the amplitude is small.

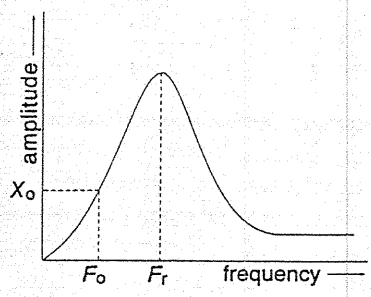


Figure 18 - *system response curve for a new motor*

Over time many things wear out: grouting turns to powder, hold-downs loosen, welds crack, and bearing clearances increase (remember that bearing lubricant is a major source of damping). The effect of wear does not change F_o; rather it causes the response curve to move. Figure 19 shows the

response curve when the system was new as a solid line, while the response curve now is shown as a dotted line. Notice that F_o has not changed, but its place on the curve has. Consequently the amplitude is much higher.

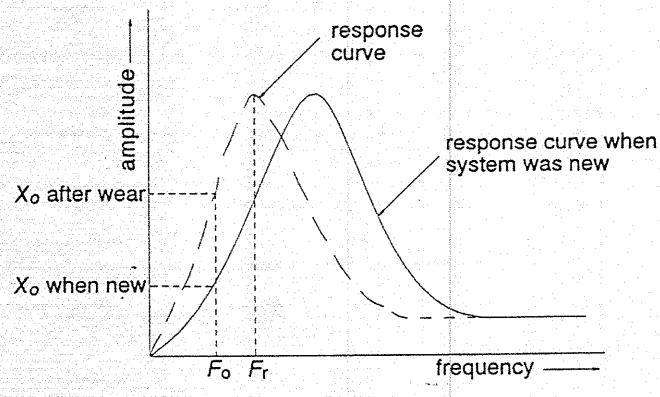


Figure 19 - *the effect of wear on system response*

The solution to this vibration problem is to maintain the system, not balance it. Regrouting, tightening hold-downs, fixing cracked welds, and replacing defective bearings will cause the response curve to move back toward its former position when the system was new.

12.3 Customer Demand

Finally, the customer may increase F_o in order to increase production. It is always tempting to increase operating speed in order to obtain more output, but all too often the increase in speed puts F_o close to F_r. Now we can state the last two rules of system response:

4. ALL SYSTEMS ARE FORCED TO VIBRATE

5. ALL SYSTEMS RUN CLOSE TO F_r

Stating that all systems are forced to vibrate may seem obvious, but many people are tempted to act like the system is not forced and to treat the symptom instead of the cause. The purpose of vibration analysis is to find and minimize the forcing function. This is preferable to stiffening and/or bracing the system to minimize the effects of vibration; better to minimize the vibration by treating the forcing function.

13.0 VECTOR REPRESENTATION OF VIBRATION

Forcing functions are forces that vary with time. If we look at our spring-mass system again, we see that the imbalance on the rotor mass changes in direction with time. As the rotor turns, the imbalance (and the imbalance force) sweeps around in a circle like the second hand on a clock. This forcing function $F(t)$ (a force varying with time) causes the spring mass system to move up and down at the frequency of the forcing function. Depending on the speed of the rotor, this up and down motion could be too fast to see. So let's point a camera at the vibrating system and, whenever the moment feels right, take a picture.

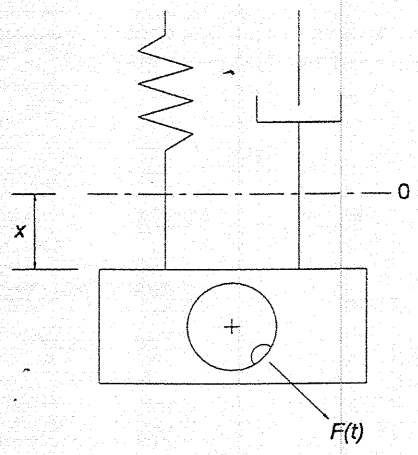


Figure 20 - *motion due to forcing function F(t)*

Figure 20 shows our "photograph" of the system. As it happened, we took a picture when the mass was moving in the negative direction, below the 0 line. The distance below the 0 line (the amplitude) is denoted by x.

Newton's laws of motion say that if an object is in equilibrium, then all the forces acting on that object at any instant in time must add up to 0. When the space shuttle is sitting on the launching pad, the force of gravity is pulling it down. But the launch pad is pushing up with the same force, so the system is in equilibrium and the shuttle just sits there.

Translation occurs when the forces become imbalanced and no longer sum to 0. When the shuttle's engines fire, the hot gases escape the rear of the shuttle at tremendous velocity. This overcomes the force of gravity and the shuttle lifts off. If any vibrating mass is in equilibrium, then all the forces acting on the mass at any instant must sum to 0. If they did not, the mass would be translating (moving). Certainly the mass is moving somewhat due to vibration (that is what amplitude and displacement are), but that motion is periodic; the mass (that is, the machine) does not translate. If it did, the vibrating machine would also be scooting across the floor!

Many of the quantities we have talked about (such as mass) can be expressed by a single number. A force, however, cannot be expressed by a single number. It is not sufficient to say that we have a force of 1 pound. We also need to say where the force is acting. Remember the force *F(t)*? It moves around in a circle, so to completely describe it we must say both how much it is and in what direction it is acting. Such quantities, which require a magnitude and a direction, are called **vectors**. Vectors are drawn as line segments, their length representing the magnitude of the force. You can pick whatever units of length you want; say 1/4 inch equals 1 pound of force. An arrowhead is drawn at one end of the line segment to indicate the direction.

If you imagine a clock face laid over our photograph (that is, Figure 20), the force *F(t)* happens to be in the 4:30 position. Instead of a clock face, we could lay x,y axes over the photograph, with the origin over the center of the rotor. We will use these axes to represent the position of the vector. Notice in Figure 21 that the end of each axis has been labeled with a number of degrees. The x axis in the positive direction is 0 degrees; if F_t is pointing directly out to the right, then its position is 0 degrees. If it is pointing straight up, then its position is 90 degrees, and so on around the rotor until we go 360 degrees and return to the 0 line.

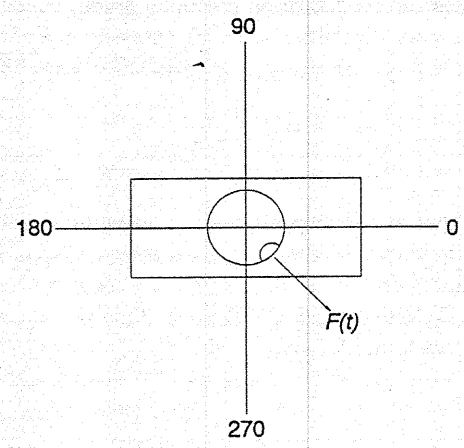


Figure 21 - *the position of F(t) on x,y axes*

It is a scientific convention when expressing angular position that the angle increases in the counter-clockwise direction. So to transfer our clock face idea to angular position, you can see that 9:00 corresponds to 180 degrees. In our photograph, the force F_t was at the 4:30 position, or 315 degrees. The industrial convention is to express phase angles as a clock face, with 0 straight up at the 12:00 position and rotation being clockwise. The vector methods discussed here use the scientific convention rather than the industrial. It does not matter which convention you use; as long as you are consistent, the solutions will be the same.

Now we can see why expressing phase in degrees is so useful. If we want to draw a vectorial representation of the imbalance force "3 lbs at 120 degrees", we can simply draw the vector as shown in Figure 22.

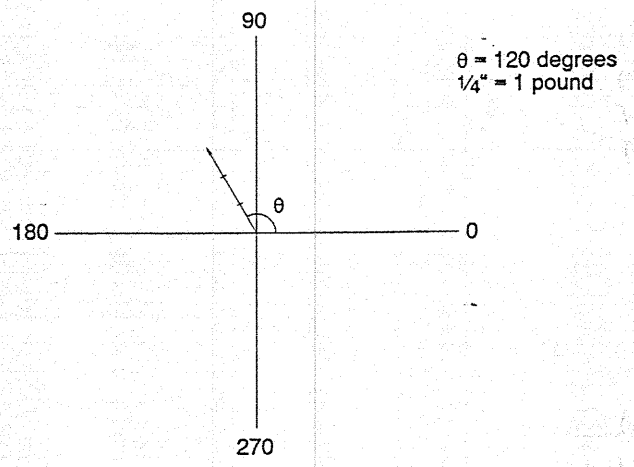


Figure 22 - *vector representation of an imbalance force*

We have used units of 1/4 inch for each pound of force. Sometimes it is helpful to place small tick marks at each unit, so you can tell at a glance that the magnitude of the vector is three pounds.

Although two or more vectors can be drawn on the same graph, in reality the vectors are added together and only their result remains. Consider two teams playing tug-of-war. The force exerted by each team can be represented by two vectors going in opposite directions, 180 degrees apart, as shown:

If teams A and B are pulling with equal strength, they are exerting equal forces and the vectors are the same length. The result of these two vectors is 0, since they are equal in magnitude but opposite in direction. The two vectors cancel each other out and neither team moves.

Now let's suppose that team B suddenly gains 5 players. Now the force that they exert is stronger than team A's, and the two vectors look like this:

The resultant of these two vectors will be a smaller force in the B direction as shown.

Team B wins.

Addition of two vectors that are other than 180 degrees apart follows the same idea, but must be done either mathematically or graphically. Figure 23 shows vector addition.

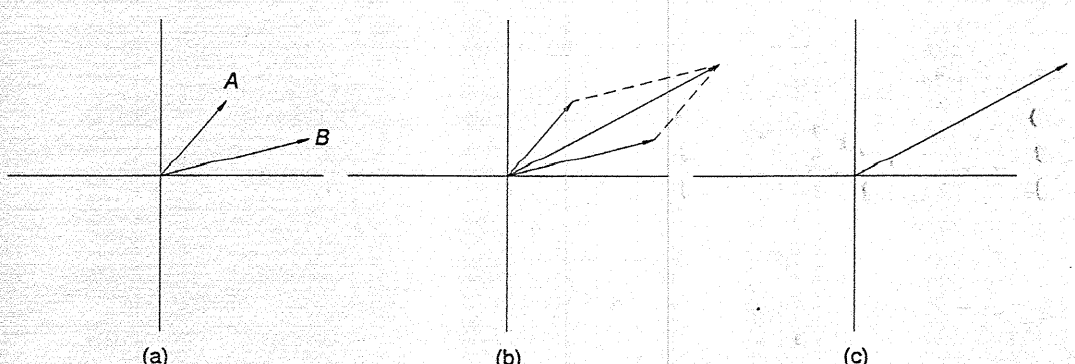


Figure 23 - *vector addition*

Two unequal vectors, labeled *A* and *B*, are shown in (a). To add the two vectors, draw a line at the tip of vector *B* equal in length and parallel to vector *A*. Repeat this for vector *B* as shown in (b). Now you have a parallelogram. If you draw a line from the origin to the opposite corner of the

parallelogram, that line is the resultant vector. Its length is equal to the magnitude of the resultant, and its angular direction is the direction of the resultant as shown in (c).

What this means is that if two forces are acting on an object as shown in (a), the object will move exactly the same as if the single force in (c) had acted on it.

Now let's return to our photograph (Figure 20) and draw a vector representation of all the forces acting on it. Since the mass is not translating, we know it is in equilibrium. Since all the forces acting on the mass are in equilibrium, we know that the vector sum of all the forces must be 0.

There are four forces acting on the mass:

1. The vertical restoring force of the spring

   $F_k = k * x$

2. The downward force from the mass and its acceleration

   $F_m = m * a$
   $= W/g * 4 * \pi^2 * f^2 * x$

3. The damping force proportional to velocity

   $F_c = c * v$
   $= c * 2 * \pi * f * x$

4. The forcing function (that is, the rotor imbalance)

   $F_f = (4 * \pi^2 * f^2 * Ub)/g$

The restoring force F_k will be acting straight up since the mass was moving down (pulling against the spring), so we will draw the vector straight up. Likewise, the downward force F_m is drawn straight down. The lengths of these two vectors represent the magnitude of the forces, and their directions are as shown in Figure 24.

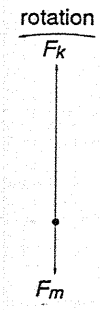


Figure 24 - *vectors F_k and F_m*

The damping force is proportional to velocity, and its magnitude is equal to velocity times damping. Since velocity leads displacement by 90 degrees, the position of this vector is 90 degrees before the displacement. The spring-mass system we have been considering here is assumed to be a single degree of freedom system, so the displacement is only up or down. The damping force vector must be 90 degrees to these two vectors, which can only be along the x axis (the line representing 0 and 180 degrees). But which side does the damping force vector go on? The right side or the left?

Remember that angular position on the x,y axes increases in the counter-clockwise direction. When you are moving around the axes in a counter-clockwise direction, you must arrive at the velocity

vector before you arrive at the displacement vector, since velocity leads displacement by 90 degrees. Since displacement was down (negative) in our photograph, displacement lies along the F_m vector. So, 90 degrees ahead of that is the vector out to the left (180 degrees), since in moving around the axes you will arrive at 270 degrees.

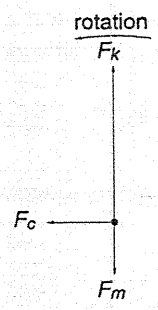


Figure 25 - *the vector representation of F_c*

Already this system doesn't appear to be in equilibrium: the F_k and F_m vectors are not equal, and there is no vector to counterbalance F_c. The only force remaining to bring this system into equilibrium is the forcing function. Since we know that the system is in equilibrium, the forcing function must adjust itself to keep the system in equilibrium. Following from Figure 25, the vector for $F(t)$ is shown in Figure 26.

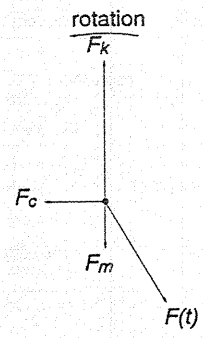


Figure 26 - *$F(t)$ brings the system into equilibrium*

If you wish, you can take the time to use the graphic method described previously to add these four vectors. You will find that all four vectors do indeed sum to 0. Now let's look at how the forcing function changes magnitude and phase to keep the system in equilibrium.

What happens if we double the motor frequency? Let's look at the effect on all four vectors. The magnitude of F_k doesn't change, since F_k equals $k * x$ (since there is no frequency in the equation, changes in frequency have no effect on the magnitude of Fk).

The damping force will change since F_c equals $2 * \pi * f * x * c$. F_c is now twice as long, since f doubled. Likewise, F_m will change since F_m depends on the square of the frequency. That vector is now four times as long. The three vectors now look like those in Figure 27.

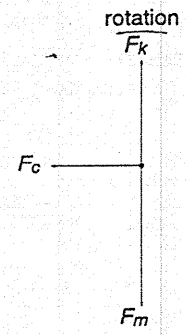


Figure 27 - *F_k, F_c, and $F_{m\bar{}}$*

For the mass to remain in equilibrium, *F(t)* must be equal to in magnitude (and in the opposite direction from) the sum of these three vectors. The vector representation of *F(t)* now looks like those in Figure 28.

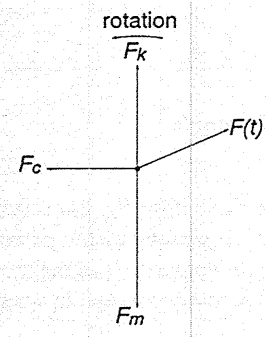


Figure 28 - *F(t) keeps the system in equilibrium*

If we increase the frequency even more, the magnitudes of F_c and F_m increase accordingly. *F(t)* then changes in magnitude and direction to be equal and opposite the sum of F_k, F_c, and F_m.

Remember that all these vector drawings are representations of the system at an instant in time. Since we changed the frequency twice, we had to take another photograph each time. For ease of demonstration we have assumed that the displacement x was constant and negative in all three cases; that way we could keep the vector for the damping force F_c on the left side. Had the displacement been positive, the F_c and *F(t)* vectors would have changed sides. The F_k and F_m vectors would also have reversed, but the summations would have remained the same.

Now let's use these vector plots to look at the behavior of the force *F(t)* as the frequency increased. Formula 2-8 showed us that imbalance force is equal to $\omega^2 * Ub/g$, where ω is equal to $2 * \pi * f$. Therefore, as frequency increased the vector for *F(t)* got longer because *f* got larger.

Also, in order to keep the system in equilibrium, the phase of the vector moved in the direction of rotation. If we took many more photographs at many different frequencies, we would see that the phase of *F(t)* would move up and down quite a bit. The phase would never be exactly 0 degrees (because of damping), and would never be 180 degrees (also because of damping).

The phase would never cross over to the same side as vector F_c. Since all systems have damping there is no way that it could; $F(t)$ must stay on the opposite side from F_c in order to oppose F_c and keep the system in equilibrium. Based on this knowledge, let's add a plot of the phase angle below the familiar frequency response plot, so that we can see how the phase varies with frequency. This is shown in Figure 29.

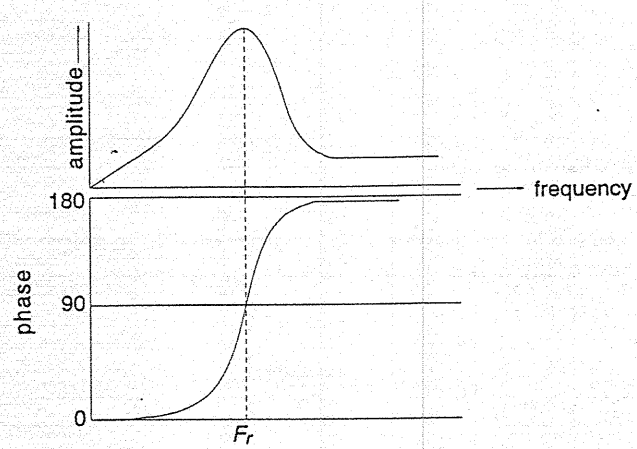


Figure 29 - *frequency response and phase angle*

This plot shows that, at frequencies far below F_r, the forcing function $F(t)$ is approximately in phase with the motion. Notice that damping prevents the phase of $F(t)$ from actually touching the 0 line. As frequency increases the phase plot begins to lead the motion, until at F_r the force is 90 degrees ahead of the motion caused by $F(t)$. This is represented in vector form in Figure 30.

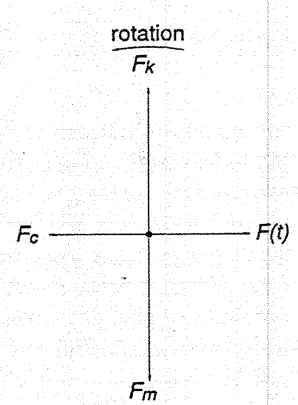


Figure 30 - *vector diagram of system running at* F_r

Notice that in Figure 30 all four vectors are 90 degrees from each other; the velocity leads the displacement by 90 degrees, and the phase of the forcing function is 90 degrees ahead of the motion.

Moving above F_r, the phase of the forcing function moves nearly 180 degrees ahead of the motion it causes.

SUMMARY

All mechanical systems have three components that affect the behavior of the system: a mass, a spring, and damping.

When a system is vibrating, the velocity leads the displacement by 90 degrees. The acceleration leads the velocity by 90 degrees and the displacement by 180 degrees. The time difference between two related events is called phase, and is measured in degrees.

The resonant frequency of a system is the frequency at which the greatest amplitude occurs for constant input of energy. It is the center of the system's amplification range, in which the energy of vibration is amplified by the system. At resonant frequency, the force leads the motion by 90 degrees. Above F_r, outside the amplification range, the force leads the motion by approximately 180 degrees.

There are five rules of system response:

1. ALL SYSTEMS HAVE DAMPING
2. ALL SYSTEMS HAVE SMALL AMOUNTS OF DAMPING
3. VIBRATORY POWER IS LOW
4. ALL SYSTEMS ARE FORCED TO VIBRATE
5. ALL SYSTEMS RUN CLOSE TO F_r, THEIR RESONANT FREQUENCY

3 TRANSDUCERS

Transducers are used to transform mechanical vibration into an electrical analog that represents (that is, is ANALOGous to) the mechanical motion. The electrical signal can then be

- amplified
- attenuated
- filtered
- integrated
- differentiated
- analyzed
- recorded

Since all vibration analysis techniques depend on this electrical signal, **the selection and mounting of the transducer is one of the most important decisions in vibration analysis.** If your data is flawed, inaccurate, noisy, or contains errors due to transducer selection, faulty electrical connection, or mounting, then even the best analysis equipment will give incorrect results; you simply cannot get good answers from bad data.

This chapter discusses the various types of transducers, their advantages and disadvantages, and offers guidelines on determining which transducer is best for each application. Mathematical techniques for converting transducer data from one set of units to another are also discussed, with appropriate cautions.

1.0 TYPES OF TRANSDUCERS

In general there are three different categories of vibration transducers, based on the type of their electrical output:

1. Displacement transducers
2. Velocity transducers
3. Acceleration transducers (called accelerometers)

A transducer of one type can be used to read vibration data of another type (for example, an accelerometer can be used to read velocity). The processes of integration and differentiation are used to convert from one type of measurement to another. Section 2.0 of this chapter discusses these mathematical conversions and some of the problems which may result from their use.

1.1 Displacement Transducers

Displacement transducers transform mechanical motion into an analog electrical signal that is proportional to the displacement (that is, the space occupied by the vibration of the system).

There are many types of displacement transducers, several of which are discussed in Section 3.0 of this chapter. The most popular displacement transducer in use today is the **non-contact eddy current probe**, shown in Figure 31.

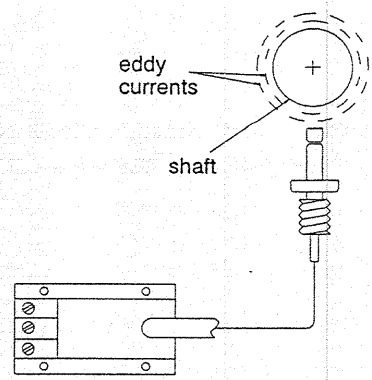


Figure 31 - *non-contact eddy current probe*

A pancake coil at the end of the transducer is excited by an external power source at a frequency somewhere between 100 kHz and 2 mHz. The high frequency field in the coil induces eddy currents (circulating electric currents) in the conducting material of the target (shaft).

Note that the target must be a conductor of electricity for the eddy current transducer to work; the transducer cannot be used on a non-conducting material. It does not need to be magnetic, only conductive.

These eddy currents cause a voltage loss in the oscillator circuit (which includes the coil). A demodulator in the power source provides an output DC signal which is proportional to the distance between the coil and the target.

If the distance between the transducer coil and the shaft remains constant, then the voltage loss also remains constant. When the shaft moves back and forth relative to the probe, the changing distance causes the strength of the induced eddy currents to change. The changing eddy currents cause the voltage out of the demodulator to change proportionally to the change in distance (called the **offset**).

The output of the eddy current transducer is a DC voltage proportional to the offset over a limited range. Since the offset changes as the shaft rotates, the DC will vary, creating a dynamic AC signal representing the change in the offset over time. This is shown in Figure 32.

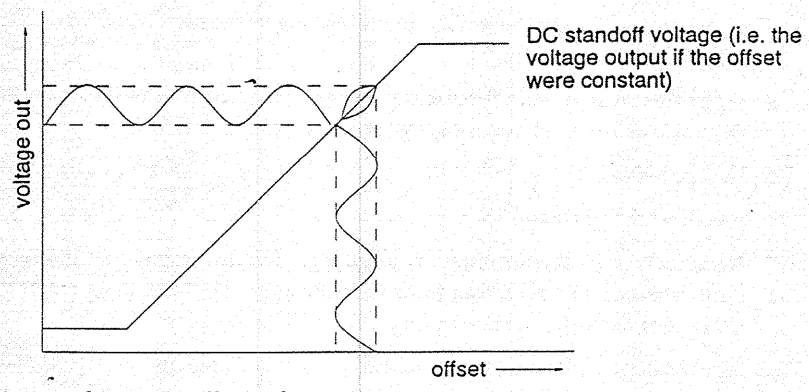


Figure 32 - *eddy current transducer DC offset voltage*

For rotors made of 4140 steel, the static standoff distance (distance between the probe and the target material) for most probes is about 30% of the probe diameter. The linear range of operation is normally plus or minus ½ of the standoff.

Eddy current transducers are used most often to measure the relative motion between the bearing journal and bearing housing of a sleeve bearing. Many times two transducers are mounted at 90 degrees to each other to see the shaft orbit. The calibration of the eddy current proximity probe includes the power source and demodulator, the lead length and capacity, and the probe itself. The system must be calibrated as a whole, and recalibrated if any of the components are changed.

The reading from a proximity probe is affected by

- shaft run-out (that is, eccentricity in the shaft's rotation)
- scratches or other variations in the shaft surface
- variations in target material
- the presence of magnetic spots left by magnetic particle inspection, grinding, or the use of a magnetic base for a dial indicator
- conducting material other than the target close to the end of the probe
- conducting particles in the gap

These probes also have temperature and pressure limitations which must be determined for critical applications.

The signal-to-noise (S/N) ratio of an eddy current transducer is difficult to estimate, since the actual S/N ratio is affected by the surface condition of the shaft, by the measurement frequency range, and by the quality of the outside power source for the excitation voltage. A reasonable estimate is from 20 to 40 Db over a frequency range from DC (0 Hz) to 1000 Hz.

Some of the advantages of the non-contact eddy current displacement transducer are as follows:

1. **Reads displacement.** This is an advantage if displacement information is what you want to measure.
2. **Measures static displacement.** Eddy current transducers are easy to calibrate; a micrometer or plastic shim stock can be used to verify the measurement readout.
3. **Measures relative motion.** The eddy current probe is ideal when you need to measure the motion between components of the machine rather than the motion in space, for example, the

movement of the shaft within a sleeve bearing or the motion of the column of a milling machine with respect to the spindle.

4. **Operates over a wide frequency range.** Eddy current transducers may be used to measure dynamic offset of shafts rotating from less than 1 to 1,000 Hz.
5. **Inexpensive.**
6. **Small and lightweight.**
7. **Research and development is ongoing.** Several transducer manufacturers are continuing their research in improving transducer design. This can only mean that the performance of these probes will continue to improve.

The disadvantages of the non-contact eddy current displacement transducer are as follows:

1. **Reads displacement.** This is a disadvantage if you need to measure velocity or acceleration data instead. As you will see in Section 2.0, differentiation is required if an eddy current displacement transducer is used to read velocity. This situation is not ideal, and a displacement transducer should never be used to read acceleration! Double differentiation enhances high frequency noise and renders the signal unusable.
2. **Requires a stable datum plane.** Since the eddy current transducer measures relative motion, the transducer itself must not be moving. For this reason eddy current transducers should not be hand-held.
3. **Requires an outside power supply.** If the power supply is noisy or drifts, then the transducer output will be noisy or will drift.
4. **Output affected by material.** Certain target materials affect the output differently; for example, the output from aluminum is nearly twice that of steel. If more than one material is present on the shaft (like a chrome-plated bearing journal), then the output will be greatly affected by the variation in chrome thickness on the steel rather than by the shaft's motion.
5. **Output affected by magnetic spots.** For example, a magnetic base for a dial indicator might have been placed on the shaft during alignment. The magnetic base leaves a magnetic spot in the shaft, which affects the vibration reading.
6. **Dynamic signal rides on DC voltage.** If you feed the transducer output into an FM tape recorder, you must first strip off the DC offset voltage (using an input capacitor or a DC bucking voltage which opposes the DC level). Otherwise this high voltage will overload the recorder and you will not record the dynamic AC information.
7. **Limited at high frequency.** Eddy current transducers are not suitable for vibration measurement of frequencies above 1,000 Hz. Normal displacements above this frequency are in the noise level of both the transducer and the instrumentation, making it impossible to read these higher frequencies accurately.
8. **Double differentiation to cross all vibration parameters.** This is the worst of all possible conditions! A displacement transducer should never be used to read acceleration; the high frequency noise enhancement introduced into the signal by double differentiation makes analysis of acceleration using a displacement transducer virtually impossible.
9. **Should be calibrated each time for the target material.** This may require a shutdown for proper calibration.

1.2 Velocity Transducers

A velocity transducer, normally a coil in a magnetic field, transforms mechanical motion into an analog signal that is proportional to the velocity of the vibration. The most common velocity transducer is the **seismic velocity transducer** shown in Figure 33.

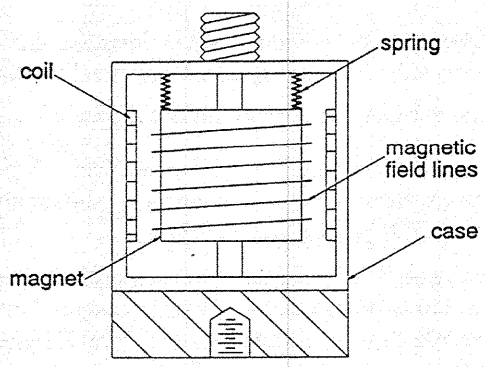


Figure 33 - *seismic velocity transducer*

A magnet is suspended inside a coil by two springs. Vibration causes a relative motion between the magnet and the coil. The magnetic field induces an electric current in the winding of the coil proportional to the speed at which the coil passes through the lines of magnetic flux. A related design suspends the coil from the spring instead of the magnet, but the performance is the same. Electrical currents are induced as the turns in the coil pass through the magnetic lines of flux. Since the magnet is a permanent magnet, a seismic velocity transducer does not require an external power supply.

Since the coil (or the magnet) is mounted from a spring, seismic velocity transducers are a typical k/m system (that is, a spring-mass system) having a resonant frequency somewhere between 2 and 10 Hz. They also exhibit the typical phase shifts of such systems when they pass through resonance. Since this spring-mass system has a high resonant amplification, these units require near critical damping.

Damping can be either magnetic, viscous (fluid), or a combination of both. Either damping method varies with temperature, so the output of the transducer changes with temperature (especially in the low frequency range). Many of these units are designed to work into a relatively low (5k to 10k ohms) input impedance; the loading serves as part of the damping so that the transducers are under-damped when working into the high input impedances of modern vibration analyzers.

The S/N ratio of the seismic velocity transducer is in the range of 60 to 90 dB, and is limited by the electronics and the presence of electromagnetic noise. The signal to noise can be further improved by using transducers designed for balanced line input to take advantage of common mode rejection (two lines are used for the vibration signal so that noise introduced into both lines is canceled out).

Seismic velocity transducers are also position sensitive: the magnet will "fall over" slightly if the transducer is mounted horizontally, causing the transducer to read differently from vertical to horizontal. Usually this difference is small (a 3 to 5% difference).

Some of the advantages of the seismic velocity transducer are as follows:

1. **Reads velocity**. This is an advantage if velocity is what you need to measure. Velocity is a good parameter to measure: it is right in the middle of all vibration parameters, requiring only

one integration or differentiation to convert to acceleration or displacement. Velocity is also proportional to the vibratory energy.

2. **Self-generating.** A seismic velocity transducer uses a permanent magnet, which induces a current in the moving coil by magnetic induction. No external power supply is required.
3. **Excellent S/N ratio.** Seismic velocity transducers have the best S/N ratio of all transducer types.
4. **Rugged.** Most seismic transducers are highly shock resistant, and can be dropped onto hard surfaces such as concrete without changing the output or calibration.

The disadvantages of seismic velocity transducers are as follows:

1. **Large and heavy.**
2. **Limited frequency range.** Seismic velocity transducers should not be used below 10 Hz or above 1,000 Hz if accurate readings are to be taken.
3. **Limited temperature range.** Most seismic velocity transducers have a limited temperature range from 0 to about 250 degrees F. More exotic designs have been made to work at temperatures as high as 700 degrees F., but their low-level frequency response is seriously limited because of the friction damping required at these temperatures. Even in the limited temperature range, the output varies more with temperature than the output of a proximity probe or an accelerometer.
4. **Some effect from magnetic fields.** The presence of strong electromagnetic fields can induce voltage in the coil, giving erroneous readings.
5. **Output varies with measurement position.**
6. **Relatively expensive.** Most eddy current probes and accelerometers cost less than seismic velocity transducers.
7. **No research to improve design.** No research to improve the product indicates the impending demise of this style of transducer.

1.3 Accelerometers

Accelerometers transform mechanical motion into an analog signal proportional to the vibration acceleration in the system. Most accelerometers use piezoelectric materials (usually quartz or polycrystalline ceramic) as the major sensing component of the transducer. Quartz is the most stable over time and exhibits the lowest change in output with temperature.

A piezoelectric material produces an electrical voltage when strained by an external force (the word piezo is derived from a Greek word meaning "to squeeze"). Atoms within the material are pushed together and electrons are displaced, forming an electrical voltage proportional to the force. Piezoelectric accelerometer transducers can be designed to operate in one of two modes: compressive mode and shear mode. Both designs utilize a mass mounted on the piezoelectric material. This assembly has a natural frequency which depends on the mass and the spring constant of the material.

The operating frequency range is usually limited to approximately 10% of the natural frequency. Units designed for high frequency (above 10 kHz) are very small, whereas units designed for low frequency (.001 to 1 kHz) are quite large and heavy. A compressive mode accelerometer is shown in Figure 34.

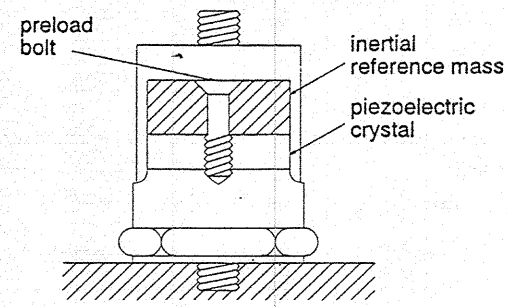


Figure 34 - *compressive mode accelerometer*

A mass is preloaded against a piezoelectric material to produce a constant baseline. When the transducer is subjected to vibratory force, the force (equal to the mass times the acceleration) adds to and subtracts from the mass preload against the piezoelectric material, producing a voltage proportional to the change in force.

In a shear mode accelerometer, the mass surrounds the piezoelectric material as shown in Figure 35. Vibratory force causes the mass to shear (that is, pull), in either direction, against the piezoelectric material. Due to the shear force against the piezoelectric material, a preload is not necessary.

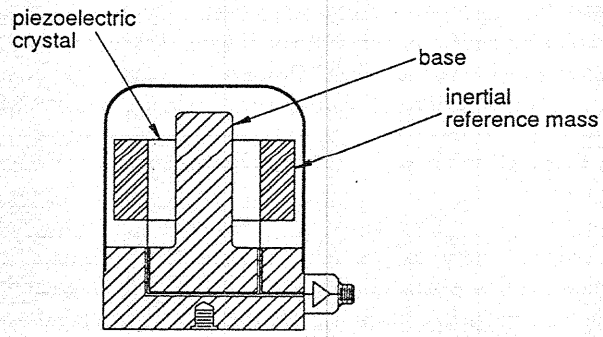


Figure 35 - *shear mode accelerometer*

Shear mode has an advantage over compressive mode in situations where base strain is put on the transducer. Figure 36 shows two machine spot faces, each with a stud for mounting a transducer to the face. Neither of these faces are level. The face shown in (a), although flat, is not normal to the stud. The face in (b) is square to the stud but is not machined completely flat: there is some small amount of foreign material on the spot face. These two illustrations are exaggerations of similar situations in real life; in reality the unevenness may be as little as 2 or 3 mils.

When the transducer is threaded onto the stud and torqued down, it contacts the spot face at an angle. As the transducer is tightened, the base of the transducer is pulled down against the face, putting strain on the base of the transducer. Base strain can distort the mass preload in a compressive mode accelerometer. This affects the transfer of force to the material and the voltage produced that is proportional to that force. With shear mode accelerometers, however, base strain has little affect on the output voltage.

Figure 36 - *poorly machined spot faces*

Some of the advantages of piezoelectric accelerometers are as follows:

1. **Reads acceleration.** This is an advantage if acceleration is what you need to measure (to detect bearing fault frequencies or gear mesh frequencies, for example).
2. **Small size.**
3. **Light weight.**
4. **Relatively inexpensive.**
5. **Wide frequency range.** Accelerometers have a very wide frequency range, and have been used up to 100 kHz under very special conditions. Frequencies of 5 to 10 kHz are normally adequate for most vibration analysis. However, if many high frequencies are present in the signal, they may drown out any low frequencies that are present. Mechanical filters between

TYPES OF ACCELEROMETERS

The voltage output of a piezoelectric material is directly proportional to the change in charge divided by the shunt capacity. By manipulating this relationship we can see that the change in charge is directly proportional to the voltage multiplied by the shunt capacity. Therefore piezoelectric accelerometers can be operated in one of two modes: voltage mode and charge mode.

In voltage mode the change in voltage is proportional to the change in force, whereas in charge mode the change in charge is proportional to the change in force.

The voltage produced by a piezoelectric material is very low and the output impedance is very high. If the accelerometer is operated in the voltage mode, the system voltage sensitivity is dependent upon the total shunt capacity across the crystal, i.e. the internal capacitance of the accelerometer plus the cable capacitance plus the input capacitance of the readout device.

This dependency upon the shunt capacitance severely limits the cable length and makes it necessary to include the cable in the calibration procedure.

Since the voltage output of the piezoelectric material is proportional to the charge sensitivity of the material divided by its internal capacitance, the accelerometer can be operated in the charge mode rather than the voltage mode. Although this eliminates the problem of capacitance attenuation as in the voltage mode, the electrical noise at the output of the charge amplifier is directly related to the ratio of the total input capacitance to the amplifier feedback capacitance.

Because of this electrical noise, expensive low-noise cable must be used to connect the accelerometer to the charge amplifier, and cable lengths are limited to reduce the shunt capacitance.

the accelerometer and the machine may be necessary to read low frequencies in the presence of high frequencies.

6. **Research to improve.** Research is ongoing to improve the performance of piezoelectric accelerometers.

The disadvantages of piezoelectric accelerometers are as follows:

1. **Reads acceleration.** This is a disadvantage if you need to use an accelerometer to read velocity or displacement. Converting accelerometer output to velocity or displacement requires integration, and integration enhances the low frequencies in the signal. Tribo effect and circuit component noise are greatly magnified by double integration when converting to displacement.
2. **High input impedance to charge amplifier.** This makes transducers without internal amplifiers sensitive to electrostatic interference, lead length, and the tribo effect.
3. **Limited S/N ratio.**

ACCELEROMETER CABLES

Shielded cable is susceptible to the **tribo effect**: noise induced in a shielded cable when it is bent. Moving a shielded cable during a vibration analysis, or stepping on one while a reading is being taken will cause electrical noise within the cable. Noise resulting from the tribo effect will be greatly magnified by double integration if an accelerometer is used to read displacement.

The length of the cable from voltage or charge mode transducers to the amplifier is also a potential problem, especially with high frequencies. As electrical signals travel through a cable, the natural capacitance within the cable causes the signals to attenuate (lessen in amplitude); the longer the cable, the more attenuation that takes place.

High frequencies attenuate more than low frequencies since the reactive impedance of the capacity in shielded input cable is much lower at the higher frequencies. A good guideline for determining cable length is:

- if high amplitude signals (greater than 10 g's) at high frequencies are present, the cable length should be less than 100 feet.
- if low amplitude signals (less than 10 g's) are present across the frequency range, the cable length should be less than 500 feet.
- if there are no high amplitude signals at high frequencies, and only low frequency signals are being measured, the cable could be up to 1000 feet in length.

The capacitance of a cable is roughly 20 to 50 picofarads per foot of cable length. As the length increases so does the total amount of capacitance. High capacitance can also cause amplifier distortion, reducing the high amplitude range and washing out low amplitude signals. A low capacitance cable (like twisted pair) is suggested for very long lengths.

These lead length and lead noise problems can be virtually eliminated by using modern accelerometers known as ICP (integrated circuit piezoelectric) accelerometers. ICP accelerometers have the electronic amplifier built into the accelerometer, effectively reducing the lead length and lead capacity to the absolute minimum. In addition, the output impedance of these units is less than 100 ohms, compared to more than 20 million ohms for most charge mode units. With this low output impedance it is possible to run very long lead lengths, greater than 1000 feet, and in some cases even use unshielded cable.

4. **Requires outside power.** The signal amplifier, whether internal or external, requires power.
5. **High frequencies can swamp element.** If many high frequencies are present along with low frequencies, the high frequencies will swamp the piezoelectric element. As a result, the low frequencies will be unreadable.
6. **Double integration required to cross all vibration parameters.** Integration enhances the low frequency noise in the signal.

1.3.1 Quartz vs. Ceramic Elements

Crystals such as quartz and tourmaline (a silicate containing aluminum and boron; also used as a gemstone) are used as piezoelectric elements. Dutch children used to play with the piezoelectric effects of tourmaline to attract ashes; they called the crystals "aschentrekkers" (ash drawers). Polycrystalline ceramic materials are also used as piezoelectric elements. Each material has its own unique advantages and disadvantages.

Changes in temperature can produce an electrical signal in a piezoelectric element (this is called the **pyroelectric effect**). A sudden, drastic change in temperature can cause a ceramic element to permanently drift from its original calibration. Also, a sudden change in acceleration has the same effect on a ceramic element's calibration.

Quartz elements are highly immune to both thermal and acceleration shock and will retain their calibration. However, accelerometers with quartz elements are usually more expensive than those with ceramic elements. With a little extra care a ceramic accelerometer will provide accurate measurement of vibration, although quartz accelerometers are recommended for long-term stability.

1.3.2 The Effect of Mounting on Response

Vibration transducers may be mounted using threaded studs, adhesives, and magnets, or they may be handheld. The objective is to use a mounting technique that provides a flat response within the frequency range being analyzed.

Stud mounting provides the best coupling to the machine and to the vibration, while adhesives, magnets, and handholding all decrease the high frequency sensitivity in varying degrees.

When using a stud mount the spot face must

- be larger than the transducer base
- be flat to within ½ mil
- have a surface finish of 10 micro inches or better
- have the stud normal (perpendicular) to the face

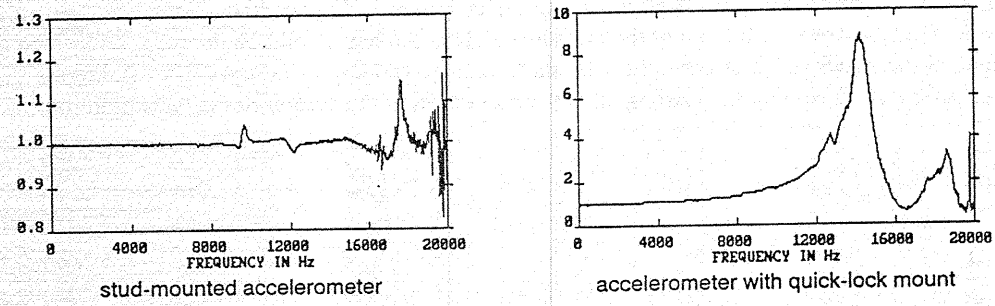
The transducer must also be properly torqued to the spot face. If you cannot ensure proper stud mounting, it is better to use an adhesive or magnet. Adhesives are a simple and effective means of mounting the vibration transducer. There are many excellent quick-setting adhesives available for mounting transducers. Consult your transducer manufacturer for their recommendations. If the surface is clean, free from grease or oil, and the product is used correctly, the transducer can be mounted quickly to most surfaces. The frequency response, greater than 6000 Hz, is adequate for most vibration analysis.

Magnets are a common and convenient means to mount vibration transducers. There are a great variety of magnets utilizing Alnico V, Alnico VIII, or one of the rare earths such as platinum cobalt or sumerian cobalt. The magnets are either ground flat for mounting on flat surfaces or made with

two narrow poles for mounting on curved surfaces. In both cases the surface must be clean and smooth enough for good magnetic adhesion, and, of course, the surface must be made of a magnetic material. If rare earth magnets are used on clean, flat, and smooth surfaces (10 micro inch finish or better), it is possible to use light-weight accelerometers up to 3,000 Hz. However, as a general rule, magnetic mounting should not be used at frequencies above 1,000 Hz.

The objective is to use a mounting technique that provides a flat response within the frequency range being analyzed. Figures 37 through 40 illustrate typical frequency response characteristics for various mounting techniques.

Figure 37 shows typical frequency response curves for stud-mounted accelerometers. The response curve on the left is for an accelerometer mounted with a stud, while the right plot shows a "quick-lock" mount. A quick-lock is a mount where a locking base is cemented to the surface and a flanged base is attached to the transducer. The two bases twist and lock together, providing good mechanical coupling with easy disconnect.



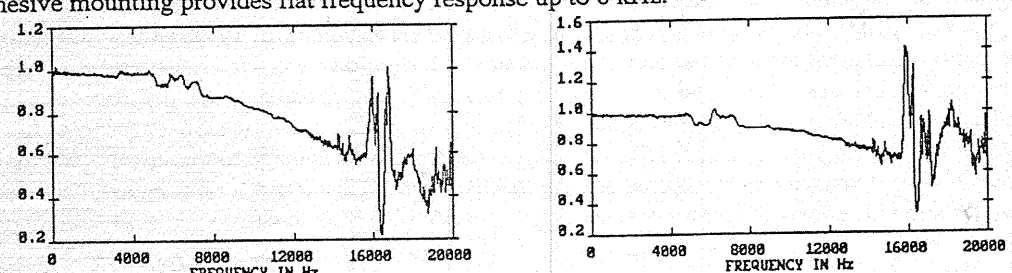
stud-mounted accelerometer

accelerometer with quick-lock mount

Figure 37 - *response curves for stud mounting*

Proper stud mounting of the accelerometer provides a flat frequency response up to 10 kHz. The spike at the high end of the curve, indicating the transducer's resonant frequency, will be located at a much lower frequency if the spot face is not machined or if the stud is not square with the surface.

The frequency response for an accelerometer mounted with adhesive is shown in Figure 38. The transducer in the left plot was mounted with a cement adhesive that sets up at room temperature in 30-40 seconds. An instant adhesive was used to secure the transducer in the plot on the right. Adhesive mounting provides flat frequency response up to 6 kHz.



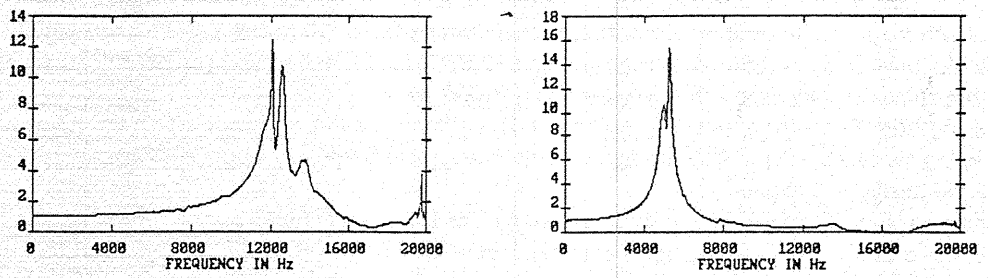
cement adhesive

instant adhesive

Figure 38 - *response curves for adhesive mounting*

Figure 39 shows the frequency response for a magnet-mounted accelerometer. If rare earth magnets are used, such as in the response curve on the left, frequencies up to 3 kHz can be measured.

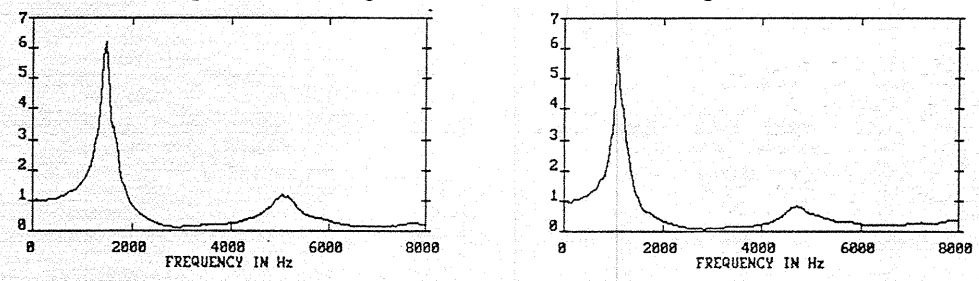
For other magnets, such as the polled magnet used in the plot on the right, the upper frequency limit is between 600 and 1,000 Hz.



rare earth magnet polled magnet

Figure 39 - *response curves for magnet mounting*

The effect of handholding a vibration transducer is shown in Figure 40. A metal rod, called a stinger, is threaded onto the bottom of the transducer. This rod has a point on the other end which is held against the machine. The point must be sharp enough to keep the stinger from sliding on the surface, but not so sharp that the loading causes deformation of the stinger or machine material.



using a 2-inch stinger using an 8½- inch stinger

Figure 40 - *response curves for hand-held accelerometers*

The actual response depends on the condition of the point, how hard you push against the machine, the stinger length, and where you place the transducer. As a general rule, the longer the stinger the lower the frequency that can be reliably measured. With care and experience, hand held transducers can be used for frequencies as high as 3,000 Hz. However, the probe has a resonance between 800 and 1,500 Hz, so the response from hand-holding is not as linear as when you use an adhesive mount.

No matter what method is used to mount the transducer it should be electrically isolated from the machine under test. Although industrial machinery is usually well grounded, it is possible to have ground currents which introduce errors in the vibration signal. In rare cases where the grounding has not been done, or has come undone, serious injury to the analyst or damage to the instrumentation may occur. Many transducers are classed as case-isolated, but the insulation is so thin that it acts as a capacitor and still allows the higher frequency ground current errors to contaminate the vibration signal.

2.0 CONVERTING TRANSDUCER OUTPUT

When you have determined the vibration units you need to measure, the ideal choice is to use a transducer that puts out a signal proportional to these units. However, if the ideal transducer is not

available or appropriate for the application, you will need to mathematically convert your transducer output into the appropriate units.

The process of integration converts acceleration into velocity or displacement. If you use an accelerometer to measure velocity, you must use integration to convert the signal proportional to acceleration to one proportional to velocity.

If a signal proportional to displacement is required, a second integration is needed to convert the signal proportional to velocity to one proportional to displacement (called **double integration**). We will explain this process by integrating the acceleration signal shown in Figure 41.

The acceleration signal in Figure 41 is represented by the formula

$$a = A * \sin(\omega t) \qquad \text{(Formula 3-1)}$$

where

a = instantaneous acceleration (in/sec^2)
A = maximum acceleration (in/sec^2)
ω = circular frequency (radians/sec)
t = time (seconds)

The circular frequency ω is equal to $2\pi f$, where f is the frequency in Hz.

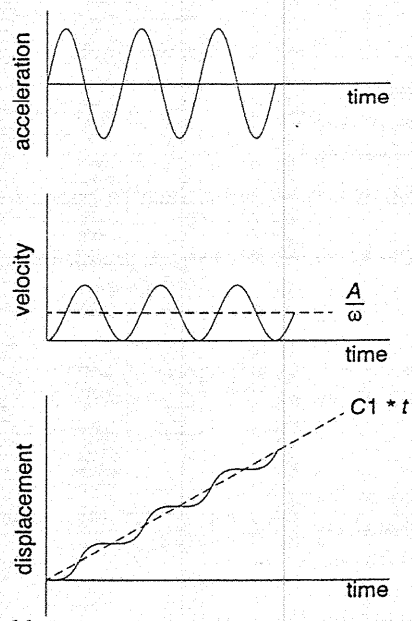


Figure 41 - *integration and double integration*

The acceleration signal is integrated to produce the velocity signal, which is represented by the formula

$$v = \frac{-A}{\omega} * \cos(\omega t) + C1 \qquad \text{(Formula 3-2)}$$

where

v = instantaneous velocity (in/sec)
A = maximum acceleration (in/sec^2)
ω = circular frequency (radians/sec)
t = time (seconds)
C1 = the integration constant

Notice that the velocity signal in Figure 41 does not go below zero; the integration has offset the entire signal by *C1*, the integration constant. To find the offset of the velocity signal we must solve for the constant *C1* in Formula 3-2. This is done by setting *t* equal to 0, since:

- at time 0 the velocity was 0, so we can set *v* equal to 0 as well
- setting *t* equal to 0 causes us to take the cosine of 0, which is 1

Setting $t = 0$ and $v = 0$ in Formula 3-2 gives us:

$$0 = -\frac{A}{\omega} * 1 + C1$$

Simplifying, solving for *C1*, and multiplying both sides by -1 gives us

$$C1 = \frac{A}{\omega} \qquad \text{(Formula 3-3)}$$

where

C1 = the integration constant
A = maximum acceleration (in/sec^2)
ω = circular frequency (radians/sec)

Formula 3-3 shows that the integrated signal proportional to velocity, taken from an accelerometer, is offset by the DC level of A/ω.

If we integrate the velocity signal again, we get a signal proportional to displacement represented by the formula

$$d = \frac{-A}{\omega^2} * \sin(\omega t) + (C1 * t) + C2 \qquad \text{(Formula 3-4)}$$

where

d = instantaneous displacement (inches)
ω = circular frequency (radians/sec)
A = maximum acceleration (in./sec^2)
t = time (seconds)
C1 = first integration constant (equal to A/ω)
C2 = second integration constant

If we solve Formula 3-4 for *C2* when $t = 0$ and $d = 0$, we find that $C2 = 0$. This means that the double integrated signal proportional to displacement is continually offset by the DC level of $(C1 * t)$.

The DC offset of the velocity signal and/or the increasing offset of the displacement signal must be removed in order to read the peak velocity or the peak-to-peak displacement. The simplest way

is to feed the signal into the analyzer through an input capacitor. Most of the time, however, an operational amplifier is used in place of (or in addition to) the capacitor to further reduce the low frequency enhancement of the integration.

You can see from Formula 3-4 that low frequency changes in the acceleration are divided by ω^2 in the double integration. If the frequency response of the system were as low as .01 Hz and low frequency tribo noise or noise from the electronic circuitry were present in this range, the division by ω^2 is equivalent to multiplying by 10,000. Dividing by ω^2 dramatically amplifies the low frequency noise, resulting in high erroneous displacement readings in the low frequency region of the vibration spectra. This integration error is sometimes called the "ski slope" since the readings are very high and taper off sharply as the frequency increases.

When double integrating acceleration to displacement, it is best not to integrate signals below 1 Hz or below 1% of the frequency range. For charge amplifier and high impedance accelerometers this should be around 10 Hz; for accelerometers with internal amplifiers this setting can be lowered to 1 or 2 Hz.

Accelerometers (particularly high impedance and charge amplifier systems) are more sensitive to electrical and triboelectric noise. Since this noise is in the low frequency range, charge and high impedance accelerometers should not be used below 5 to 10 Hz.

Differentiation is the reverse of integration; where integration took us from acceleration to displacement, differentiation converts a displacement signal into velocity or acceleration data. Figure 42 shows a displacement signal converted first to velocity by differentiation and then to acceleration by double differentiation.

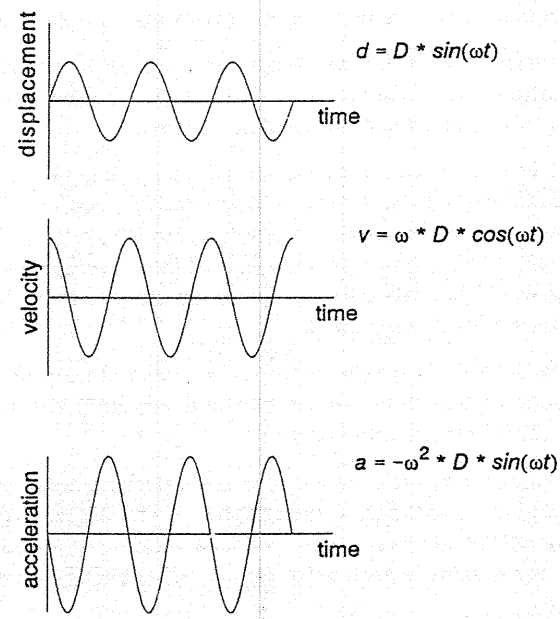


Figure 42 - *differentiation and double differentiation*

Converting a displacement signal to one proportional to velocity or acceleration causes no errors due to low frequency noise and has no DC offsets due to integration constants. However, differenti-

ation enhances the high frequency electronic and process noise. Since vibration also occurs at the higher frequencies, it is important that the signals be as undistorted as possible. Therefore, instrumentation designers prefer to integrate rather than differentiate.

Your choices for transducer selection are, in order of preference:

1. Use a transducer whose output is proportional to the vibration units you want to measure.
2. Integrate once, with the integration cutoff at 1 to 5 Hz.
3. Differentiate once, with the high frequency cutoff at 500 to 1,000 Hz.
4. Integrate twice, with the integration cutoff at 2 to 10 Hz.

It would seem that there should be a fifth choice: differentiate twice with a cutoff around 1,000 Hz. However, the high frequency distortion caused by double differentiation is too severe, and renders the signal useless for vibration analysis. **Never use a displacement transducer to measure acceleration!**

3.0 OTHER TYPES OF TRANSDUCERS

In addition to the displacement, velocity, and piezoelectric accelerometers discussed in this chapter, there are special vibration transducers available. Some of these include

- strain gauge accelerometers: have very low output impedance so that many of the lead capacity and lead length problems discussed in Section 1.3 are greatly reduced. Strain gauge accelerometers operate with output impedances similar to ICP piezoelectric accelerometers.
- accelerometers with internal integration: the output is proportional to velocity instead of acceleration. Since the integrator is built into the accelerometer itself, the lead length and capacity are reduced to their absolute minimum.
- triaxial accelerometers: a single unit which reads acceleration in three directions: normally horizontal, vertical, and axial. These transducers can be very convenient in studying structures and mode shapes, and in deriving empirical data for finite element analysis. In vibration analysis, however, only one of the three directions is normal to the shaft in the bearing where the data is being taken; the other two directions will have both a rotational and a translational component.
- transducers with temperature probes: a temperature probe is included in the base of the transducer so that both temperature and vibration can be read at the same time if the transducer is permanently installed.
- LVDT (linear variable differential transformers): a transformer with the core mounted on a spring and two secondary windings phased 180 degrees opposite to each other. When the core is exactly between both secondary windings the induced voltage is zero (since the voltage is the same in each secondary winding but 180 degrees out of phase).

  As the core moves closer to one winding the induced voltage in that winding increases and the voltage in the other winding decreases so the output is no longer zero. The phase of the output voltage indicates which direction the core is moving, and the voltage indicates how far. LVDTs require an external power supply and can be used as displacement transducers; however, their use in vibration analysis is very limited.

- variable resistance and capacitance: transducers with variable resistance are relatively inexpensive but are very temperature sensitive. Variable capacitance transducers are very light and can be designed for non-contact sensing, but they are hard to keep in calibration.

In addition, microphones can be very useful in analyzing high frequency vibration problems such as gear mesh frequencies and noise in vibrating systems such as air ducts and piping.

SUMMARY

Transducers are used to transform mechanical vibration into an electrical analog that represents the mechanical motion. The selection and mounting of the transducer is one of the most important decisions in vibration analysis.

There are three different categories of vibration transducers, based on their electrical output:

1. Displacement transducers
2. Velocity transducers
3. Acceleration transducers (called accelerometers)

Displacement transducers transform mechanical motion into an analog electrical signal proportional to the displacement. The most popular displacement transducer in use today is the non-contact eddy current probe.

A velocity transducer, normally a coil in a magnetic field, transforms mechanical motion into an analog signal proportional to the velocity of the vibration. The most common velocity transducer is the seismic velocity transducer.

Accelerometers transform mechanical motion into an analog signal proportional to the vibration acceleration in the system. Most accelerometers use piezoelectric materials as the major sensing component of the transducer.

Transducers may be mounted by studs, adhesive, magnets, or may be handheld. Stud mounting provides the best coupling to the vibration, while the other methods decrease the maximum frequency that can be effectively read.

A transducer of one type can be used to read vibration data of another type. The processes of integration and differentiation are used to convert data from one type of measurement to another. Both processes distort the vibration signal: integration amplifies low frequency noise, while differentiation enhances the high frequency electronic and process noise.

Choices for transducer selection are, in order of preference:

1. Use a transducer whose output is proportional to the vibration units you want to measure.
2. Integrate once, with the integration cutoff at 1 to 5 Hz.
3. Differentiate once, with the high frequency cutoff at 500 to 1,000 Hz.
4. Integrate twice, with the integration cutoff at 2 to 10 Hz.

Never use a displacement transducer to measure acceleration!

4

GENERAL ANALYSIS TECHNIQUES

All mechanical systems are forced to vibrate by one or more forcing functions (see the System Response chapter for a complete definition of **forcing function**). In some cases this force may be small, and natural and/or designed sources of damping within the system keep the vibration within acceptable levels.

However, a forcing function may cause a system to vibrate excessively. This vibration can result in bearing failure, fatigue and cracking in machine components, and unacceptable damage to the product being produced. Left uncorrected, excessive vibration can destroy entire structures and result in serious injury or death.

In essence, a vibration problem is solved by identifying the forcing function(s) and eliminating or minimizing their effect. While this is not always as simple as it may sound, a general step-by-step procedure may be followed during analysis of a vibration problem. This section provides an outline of this procedure and discusses a variety of analysis techniques that can aid in vibration problem solving.

1.0 OUTLINE TO VIBRATION PROBLEM SOLVING

The analysis of a vibration problem can be broken into seven steps:

1. Identify the problem.
2. Gather information about the vibrating system.
3. Determine the possible forcing function(s).
4. Determine where to take data and what equipment to use.

5. Take vibration data.
6. Analyze vibration data.
7. Make recommendations.

1.1 Identify the Problem

Not all machine problems attributed to vibration are, in fact, vibration. Noise is quite often mistaken as vibration. Noise may be created, for example, by the way electrical conductors are dressed (laid) in a raceway. The electrical field in the conductors causes them to vibrate and impact the raceway, creating a very annoying noise. This noise is not a vibration problem that can be solved by correction methods such as balancing or alignment.

Another common source of noise is the process itself, such as the flow of air in an air duct. In many cases air ducts are rather small, making the air velocity quite high. This high velocity can cause a very significant noise, one that is quite often disturbing to the occupants of the building. Correction methods used for vibration cannot be used to eliminate this noise. One solution would be to slow down the velocity of the air, but this would reduce the amount of air available for heat, air conditioning, and exhaust. The solution to this noise problem is usually to install some kind of noise isolating material around the ducts or inside the walls.

In general, noise is solved by isolation and insulation, while vibration is solved by correction. Isolating material is often the only solution for certain kinds of noise problems such as packages sliding down chutes at a routing hub of a major package shipper. Since the chutes must be used to move packages from one zone to another, the only solution to the noise is to insulate the chute or provide ear protection for the sorters.

When approaching a vibration problem, the first step is to determine if the problem is actually vibration; if the problem is noise, then the vibration analysis techniques discussed here will not be effective.

Sometimes noise and vibration are combined. For example, a piece of metal may be loose inside a paper roll. As the roll turns the metal bangs around inside, causing noise and vibration.

1.2 Gather Information About the Vibrating System

The next step in the solution of a vibration problem is to gather as much information as possible about the machine you are going to analyze. There are three essential steps in this information gathering process:

1. Sketch the machine and collect all component information (bearings, belts, gears, etc.).
2. Gather the available maintenance history.
3. Obtain the machine operator's input.

1.2.1 Sketch the Machine

A sketch of the machine allows you to determine the best data collection points for your analysis, and aids you in determining potential sources of vibration.

Many times, without a sketch, you may assume that horizontal, vertical, and axial readings at every bearing are the only readings that need to be taken. Once you make the sketch, however, it becomes apparent that other readings should be taken: on the shaft, looking down on the foundation, motor

feet, piping, etc. Once you've sketched the machine it is much easier to decide where to take vibration data.

Figure 43 shows a sample sketch of a mechanical system prior to vibration analysis.

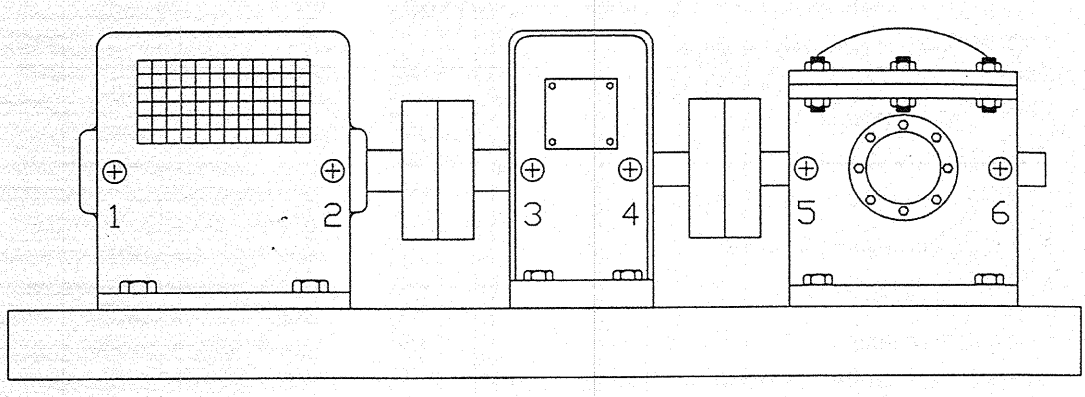


Figure 43 - *machine sketch for vibration analysis*

In this sketch we see that the system has six main components: a motor, a gear box, a pump, two couplings, and a foundation. The shafts inside these three components are each supported by two bearings. Therefore there are a total of six bearings within the system. Each bearing is shown as ⊕ in the drawing, and the bearings are numbered 1-6 for easy reference. Bearings are a good place to begin taking vibration readings: they are usually accessible since they are outside major machine components, and the bearing housing usually provides a good transmission path from the vibration source.

Do not attempt to measure vibration on flexible metal panels which may appear to be solid members of the structure. For example, most electric motors have a plate that covers the outboard rotor bearing. This plate is thin and flexible, and axial readings taken on this plate may be amplified by the resonant frequency of the plate.

To ensure consistency, readings should always be taken from the same place during a vibration survey. If you are analyzing a machine for changes, your analysis will be more difficult if the data is not collected from the same point each time. By sketching the machine and labeling each collection point, you can be sure that you collect data from the same point each time. It is also helpful to mark the exact collection point on the bearing housing by methods such as painting a mark, making a drill dimple, gluing on a magnetic mount, or installing permanent studs or quick disconnects. Your sketch should identify

I. All the major components

II. Shaft diameters and lengths

III. Rotor dimensions and weights

IV. All the bearings in the machine

V. The bearing type, number, and specifications

A. Ball (antifriction, rolling element) bearings
   1. ball or roller diameter
   2. number of balls or rollers
   3. pitch diameter (between ball centers across bearing)
   4. contact angle between ball and race
   5. shaft speed in revolutions per second (RPM/60)

B. Sleeve (journal) bearings
   1. type of sleeve bearings
   2. clearance

VI. Belts and chains
   A. Center to center distance
   B. Pulley pitch diameters
   C. Number of belts
   D. Belt length
      1. can be calculated (see Section 1.3.3)

VII. Couplings
   A. Types
      1. gear
      2. disc
      3. bun
      4. fluid
   B. Bending tolerance vs. coupling length

VIII. Drives
   A. Motors
   B. Engines
   C. Turbines

IX. Gears
   A. Gear type and layout
   B. Number of teeth
   C. Gear ratio
      1. # teeth of driven/# teeth of driver
   D. Speed of drive gear

A sample collection form used by the authors is shown at the end of this chapter.

1.2.2 Gather the Available Maintenance History

The maintenance history of the machine often provides valuable clues to the cause of vibration. Among the things you should determine are:

- What was the last thing done to the machine? If, while researching the maintenance history of a vibrating machine you discover that bearings have been replaced several times due to premature failure, then you can suspect
  - poor installation techniques
  - inadequate lubrication procedures
  - bearing misalignment
  - bearing load
  - operating frequency
  - shaft misalignment
  - electrical discharge through the bearing

  as the cause of vibration.
- What is the maintenance history of the machine? Has it ever been damaged? Have major components (bearings, gears, couplings, belts, rotors, etc.) been replaced or repaired? Has the suspect component had a history of problems?
- How thorough has general maintenance been? Has lubricant been maintained and changed regularly?
- Bearings: How often are clearances checked, and are they still in tolerance? Have any bearings been changed? If so, was the change due to normal wear or premature failure? Do the bearings need to be lubricated? If so, have they been lubricated regularly and properly? Over-lubrication is as serious a problem as under-lubrication. You should also determine which bearings are fixed and which are floating. There should only be one fixed bearing on a shaft. More than one fixed bearing on a single shaft can lead to early bearing failure.
- Shafts: What is the shaft material, and has any work been done on the shaft? If the shaft is made of a high-stress material like cold-rolled steel it will stress relieve. If a machine with a shaft of high-stress material is shut off for any length of time, the shaft will bend in the direction of the static load, necessitating a slow roll of the shaft before the rotor is brought up to normal operating speed.
- Alignment: How were the shafts aligned, and to what tolerance? If there are couplings on the shaft, is the alignment tolerance within the tolerance of the coupling? Was allowance made for thermal growth, and was the thermal growth checked? How often is the alignment checked? When were the shafts aligned— right at machine startup or after the machine had run for awhile?
- Check to see if there has been any previous work done on the machine to correct vibration. If so, compare the type of vibration readings taken then to what you have taken now. Similar readings could indicate that a correction weight that was previously installed has come off or that a previous problem is developing again.

1.2.3 Obtain the Machine Operator's Input

The machine operator is a good source of information on the characteristics and severity of the machine's vibration. Operators develop a "feel" for a machine when they work with it every day, and are often aware of developing problems before they become serious.

The types of information an operator can provide include the following:

- How did the vibration problem develop? Did it come on suddenly or was it gradual?

- Is the vibration continuous? If not, under what conditions does it occur (startup, operating frequency, load, temperature, time, etc.)?
- Has the vibration occurred before? If so, how often, under what conditions, and what was done to correct it?
- Has the operator noted the onset of new and/or different noises during operation?

1.3 Determine the Possible Forcing Function(s)

The determination of the forcing functions (the cause, or causes, of the machine's vibration) that are present in the system is the goal of vibration analysis. Once the forcing function has been identified, intelligent decisions can be made for correction of the problem.

If you don't know what the forcing function is then you can't know, for example, whether the problem is noise or vibration. A detailed analysis is necessary in order to identify the forcing function and decide what steps should be taken to solve the problem.

The information gathered in Section 1.2 is very useful in determining the possible forcing functions that are present. Once you decide which forcing functions are likely to be present, you can use the system data to estimate their frequency. Later, when you take actual vibration data, you compare your readings to your estimated forcing function frequencies. If they are close (or if they match!), then you have isolated a major cause of the system's vibration.

It is important to realize that the major vibration may be a harmonic (a multiple) of the forcing function. Resonances within the system can amplify one of the harmonics of the forcing function, causing the amplitude of that harmonic to exceed the amplitude at the forcing function frequency. There are many cases when the dominant vibration is labeled as the forcing function when the forcing function is, in fact, something running at a sub-multiple of that frequency.

The most common forcing functions include

- Rotor frequencies
- Bearing fault frequencies
- Rubs
- Belt frequencies
- Gear mesh frequencies
- Blade pass frequencies
- Harmonics of any of these frequencies
- Resonant frequencies

1.3.1 Rotor Frequencies

The major causes of vibration at rotor frequency include

- Imbalance in the rotor:
  - stress relief in the rotor
  - weight shift in the rotor
  - material that has flown off rotor (like a blade tip)
  - material built up at uneven locations
- A bow in the rotor shaft

- Misalignment of rotor bearings
- Shaft center line (coupling) misalignment

All of these can cause vibration at rotor frequency or at a harmonic of rotor frequency. If the rotor frequency (or harmonic) is close to one of the other resonances in the system, then that resonance will amplify the rotor forcing function energy.

1.3.2 Rolling Element Bearing Fault Frequencies and Harmonics

Rolling element (or anti-friction) bearings have low starting friction and high running friction. There is a frequency barrier which seriously limits the use of rolling element bearings in high-speed applications. For conservative designs and long life, the shaft frequency of grease-lubricated anti-friction bearings should not exceed 7,200 divided by the shaft diameter. Shaft frequencies for oil-lubricated (or oil mist) bearings should not exceed 9,600 divided by shaft diameter.

For example, a 6-inch diameter shaft mounted in a grease lubricated anti-friction bearing should not be run at a frequency higher than 1,200 RPM (20 Hz). These numbers are conservative, but they should be followed if bearing life is critical in the production process.

Rolling-element bearing fault frequencies are generated as a result of fatigue, wear, improper installation, improper lubrication, and manufacturing faults in the bearing components. Figure 44 illustrates the four components of a rolling-element bearing: the cage, the inner race, the outer race, and the rolling elements. Figure 44 also shows the pitch diameter *Pd*, which is the distance between the centers of two opposite rollers.

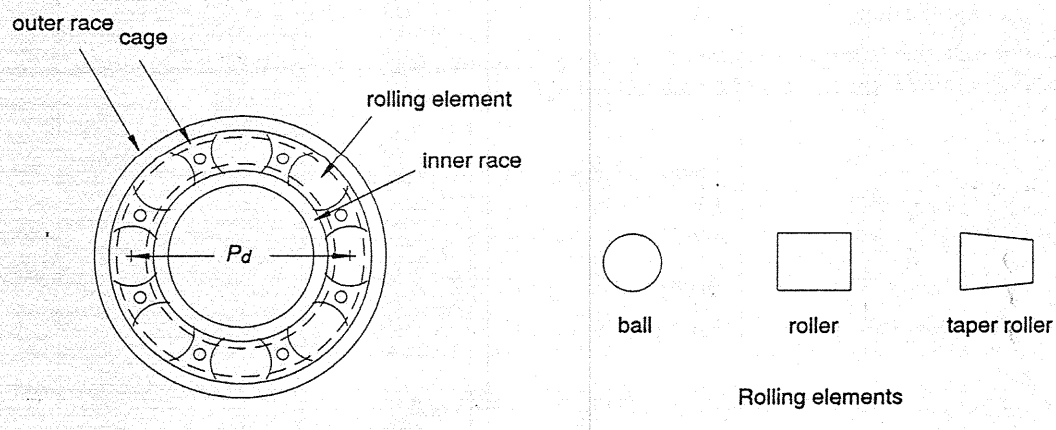


Figure 44 - *anti-friction bearing components*

Since no bearing is perfect, all rolling element bearings will have faults such as irregularities in the surface of the race and in the roundness of the balls. When a bearing spins, these faults create periodic frequencies called **fundamental defect frequencies.**

There are four fundamental defect frequencies in rolling element bearings:

FTF = Fundamental Train (or cage) Frequency.

BPFI = Ball Pass Frequency of the Inner race. This is the frequency created as all the balls roll across a fault in the inner race.

BPFO = Ball Pass Frequency of the Outer race. This is the frequency created as all the balls roll across a fault in the outer race.

BSF = Ball Spin Frequency. This is the circular frequency of each ball as it spins during revolution around the shaft.

Fundamental defect frequencies depend on both the bearing geometry and the shaft speed. If you know what type of bearings are in the machine, you can use manufacturer data to find the ball diameter, pitch diameter, number of rolling elements, and the angle between the surface of each rolling element and the races (called the **contact angle**). Knowing these three values allows you to calculate the four fundamental defect frequencies accurately.

There are situations, however, where the bearing fault frequencies that appear in the vibration spectra do not match your calculated frequencies. The most common reason for this variation is that an unanticipated load in the bearing changes one of the parameters used in your calculation. Typically the parameter that changes is the contact angle.

Unanticipated thrust loads can occur from many sources, such as magnetic center misalignment in an electric motor. In certain types of bearings (such as zero contact angle bearings or bearings rated only for pure radial load), this thrust load causes the bearing to run at a different contact angle than its normal contact angle.

A bearing can be mounted and used in two ways:

1. The inner race is stationary and the outer race is rotating. This is often used in the front wheels of automobiles.
2. The inner race is rotating and the outer race is stationary. This is the most common industrial application.

For case one (inner race stationary, outer race rotating), the fundamental fault frequencies *BPFI*, *BPFO*, and *FTF* are calculated by the following formulas:

$$BPFI = \frac{Nb}{2} * S * \left(1 - \frac{Bd}{Pd} * \cos\theta\right) \qquad \text{(Formula 4-1)}$$

$$BPFO = \frac{Nb}{2} * S * \left(1 + \frac{Bd}{Pd} * \cos\theta\right) \qquad \text{(Formula 4-2)}$$

$$FTF = \frac{S}{2} * \left(1 + \frac{Bd}{Pd} \cos\theta\right) \qquad \text{(Formula 4-3)}$$

where

BPFI = Ball Pass Frequency of Inner Race (Hz)
BPFO = Ball Pass Frequency of Outer Race (Hz)
FTF = Fundamental Train Frequency (Hz)
Nb = the number of balls
S = speed (revolutions per second)
Bd = the ball diameter (in. or mm)
Pd = the pitch diameter (in. or mm)
θ = the contact angle (degrees)

For case two (inner race rotating, outer race stationary), the fundamental fault frequencies *BPFI*, *BPFO*, and *FTF* are calculated by

$$BPFI = \frac{Nb}{2} * S * \left(1 + \frac{Bd}{Pd} * \cos\theta\right) \quad \text{(Formula 4-4)}$$

$$BPFO = \frac{Nb}{2} * S * \left(1 - \frac{Bd}{Pd} * \cos\theta\right) \quad \text{(Formula 4-5)}$$

$$FTF = \frac{S}{2} * \left(1 - \frac{Bd}{Pd}\cos\theta\right) \quad \text{(Formula 4-6)}$$

where

BPFI = Ball Pass Frequency of Inner Race (Hz)
BPFO = Ball Pass Frequency of Outer Race (Hz)
FTF = Fundamental Train Frequency (Hz)
Nb = the number of balls
S = speed (revolutions per second)
Bd = the ball diameter (in. or mm)
Pd = the pitch diameter (in. or mm)
θ = the contact angle (degrees)

The formula for *BSF* is identical for both cases:

$$BSF = \frac{Pd}{2Bd} * S * \left[1 - \left(\frac{Bd}{Pd}\cos\theta\right)^2\right] \quad \text{(Formula 4-7)}$$

where

BSF = Ball Spin Frequency (Hz)
Pd = the pitch diameter (in. or mm)
Bd = the ball diameter (in. or mm)
S = speed (revolutions per second)
θ = the contact angle (degrees)

If you don't know the geometry of the bearings installed in the machine, then you will be unable to determine the exact fundamental defect frequencies of the bearings. However, you can estimate *FTF*, *BPFI*, and *BPFO* if you know the number of balls and the RPM of the rotor:

$$FTF \approx \left(\frac{1}{2} - \frac{1.2}{Nb}\right) * RPM \quad \text{(Formula 4-8)}$$

$$BPFI \approx \left(\frac{Nb}{2} + 1.2\right) * RPM \quad \text{(Formula 4-9)}$$

$$BPFO \approx \left(\frac{Nb}{2} - 1.2\right) * RPM \quad \text{(Formula 4-10)}$$

where

Nb = the number of balls

These estimates are usually accurate to within 5-10% of the actual frequencies. In addition, when you know the type of equipment you can approximate the number of balls. Most electric motors have between 8 and 12 balls or rollers, while larger machine bearings may have as few as 10 to as many as 30 elements per row.

Ball spin frequency cannot be estimated without detailed information on the bearing's geometry. But ball spin frequency is rare; most vibration analysts never see a fault at a bearing's ball spin frequency. The most common bearing problem is outer race fault in the load zone; inner race faults are the next most common.

1.3.3 Sleeve Bearings

Rotating elements are not used in sleeve bearings; rather, the shaft rides on a layer of lubricating oil inside the bearing journal. The lubricant may be sealed inside the bearing, gravity fed to the bearing, or pumped in (called a **pressure fed** sleeve bearing).

Pressure fed bearings are used for heavily loaded shafts that start and stop often (like a large electric motor driving an intermittent process). To prevent the shaft rubbing against the bearing material (called **wiping**), lubricant is introduced under relatively high pressure in order to hydrostatically float the shaft before startup. The lubrication is put under pressure 20 minutes to 1 hour before startup to insure that the shaft is floating on the oil. Figure 45 illustrates a shaft rotating in the center of a sleeve bearing.

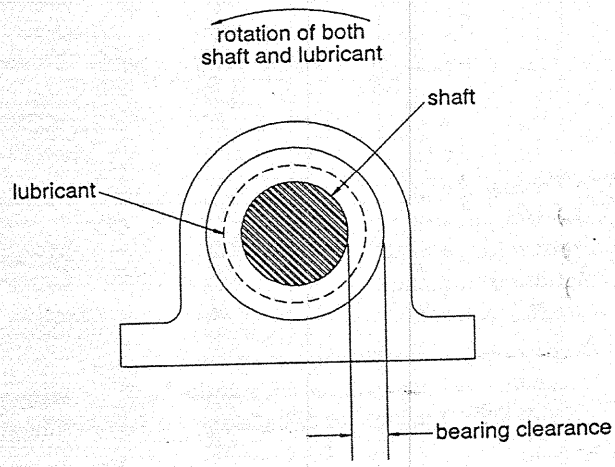


Figure 45 - *sleeve bearing*

The space between the shaft and the bearing housing is called the **bearing clearance**. Sleeve bearing clearance should never be less than 2 mils, and should be at least 1 to 1.5 mils for every inch of shaft diameter. For example, a 6-inch diameter shaft should have a clearance of 6 to 9 mils. The clearance has an effect on the system's vibration, since damping increases as clearance decreases.

Sleeve bearings have high starting friction and low running friction. Figure 46 shows a plot of friction vs. RPM. At low RPM the friction between the shaft and the oil is quite high in the area called the boundary layer. The friction begins to decrease as the shaft frequency passes through the mixed lubrication area. When the shaft frequency is in the fluid film area, friction is at a minimum.

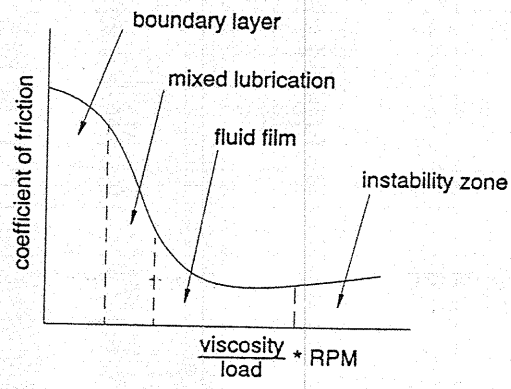


Figure 46 - *friction vs. RPM*

Sleeve bearings should always operate in the fluid film area when the shaft is at operating frequency. Three major factors determine the threshold of the fluid film layer:

1. Frequency of the shaft
2. Viscosity of the lubricant
3. Load on the bearing

The operating frequency, the viscosity of the lubricant, and the bearing load are all important design considerations for sleeve bearings. It is important that the bearing size, shaft speed, and lubricant combinations be correct so that the unit runs in the fluid film area during normal operating conditions. One of the major problems encountered in pressure fed sleeve bearings is the possibility of hydraulic instability of the shaft inside the bearing, called **oil whirl** or **oil whip**.

The rotational velocity of the lubricant next to the shaft will be equal to that of the shaft, whereas the rotational velocity of the lubricant next to the bearing surface will be zero. If the shaft were perfectly centered in the bearing, and the friction losses were ignored, the average rotational velocity of the lubricant would be exactly 50% of the shaft frequency. However, the frequency runs somewhere between 35 and 49% of the shaft rotational frequency due to friction losses and shaft eccentricity.

If the shaft is moved off center due to load, eccentricity, or imbalance, then the clearance on one side of the bearing will be greater than it is on the other side. Figure 47 illustrates this condition.

As the lubricant rotates at approximately 50% of shaft frequency it must squeeze through the narrow area where the shaft is closest to the bearing. The average velocity of the lubricant decreases as the lubricant enters the narrow gap, speeds up inside the gap, and slows down again when it leaves the gap. This speeding up and slowing down creates turbulence on both sides of the gap, and a vortex develops in the high-pressure lubricant zone.

The shaft rides on this oil vortex much like a surfboard rides the surface of a wave. Called oil whirl, this causes the shaft to become unstable at somewhat less than half its rotational frequency.

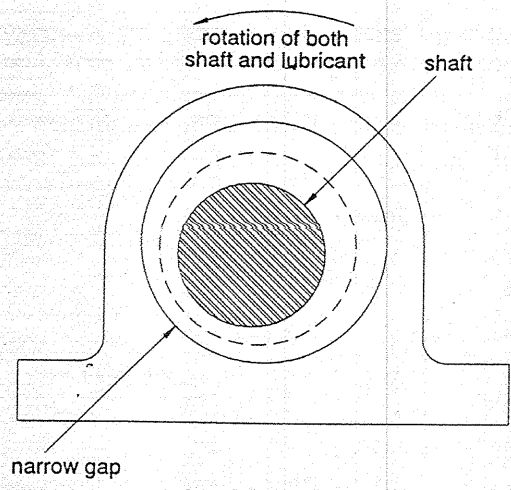


Figure 47 - *oil whirl*

The oil whirl stays proportional to the shaft frequency and drops out when shaft RPM drops below the instability threshold.

Oil whirl frequency approaches the first critical of the shaft as the shaft approaches its second critical, creating a resonant condition called oil whip. The oil whip frequency remains constant at the first critical of the shaft and drops out when shaft frequency drops below the second critical. Both of these phenomena can be severe and result in a penetration of the lubricant film. When this happens, the shaft impacts against the bearing journal and serious damage can result.

One solution is to change the loading, lubricant, temperature, or clearance so that the shaft does not run in this instability zone. Some sleeve bearing designs include axial grooves or offset cylindrical grooves in the journal to break up the oil whirl (see Figure 48).

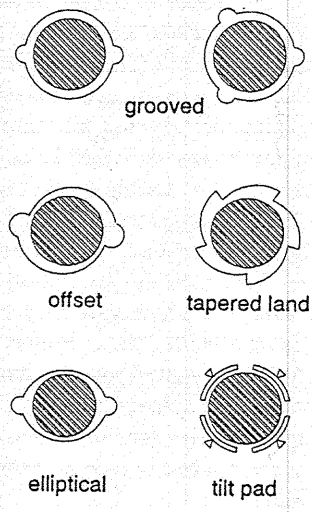


Figure 48 - *bearing designs that prevent oil whirl*

The tilt-pad bearing is the preferred solution for oil whirl or oil whip. This type of bearing should not be base loaded, i.e. should not be installed so that the shaft sits on one of the pads. Hydraulic instability in tilt-pad bearings is very rare.

1.3.4 Rubs

Rub is a general term for seal rubs, non-uniform restrictions to shaft movement, and anti-friction bearings loose in their housing. All of these problems result in ½ or ⅓ order components in the frequency spectra and a non-symmetrical display in the time domain or orbit data.

1.3.5 Belt Frequencies

The **primary belt frequency** is the number of times per minute that the belt makes one complete circuit. This frequency is found by

$$Fb = \frac{\pi * Sd * RPM}{Bl} \qquad \text{(Formula 4-11)}$$

where

Fb = primary belt frequency (Hz)
Sd = sheave diameter (in. or mm)
RPM = the speed of that sheave (CPM)
Bl = the belt length (in. or mm)

For example, the belt frequency Fb for a 150-inch long belt over a 20-inch diameter pulley spinning at 1800 RPM is equal to:

$Fb = (\pi * 20 * 1800)/150$
$= 113{,}097.34/150$
$= 753.98$ CPM

A belt rotates around at least two sheaves (pulleys); serpentine drives have more than two sheaves. It does not matter which sheave you use in this formula, since the peripheral velocity of all the sheaves within the drive must be equal (if they weren't, the sheaves would slip beneath the belt).

To understand this, look at Figure 49. Two pulleys, labeled *A* and *B*, are connected by a single belt. Pulley *A* has a diameter of 24 inches, while pulley *B* has a diameter of 12 inches. Therefore the circumference of *A* is twice that of *B*.

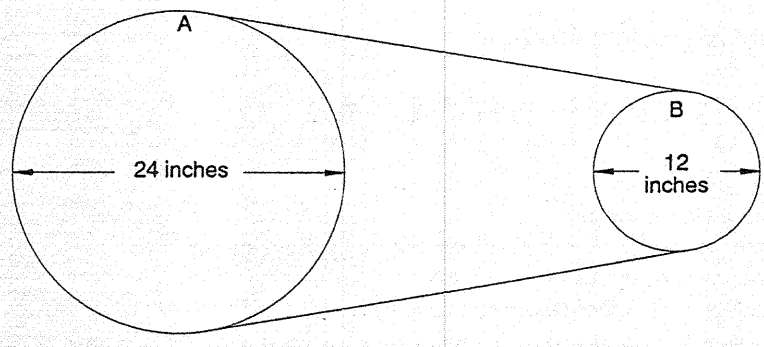


Figure 49 - *peripheral velocity of two sheaves*

Now let's connect the two pulleys by a belt 100 inches long and drive pulley *A* at 1 cycle per minute. Since *B* is half the circumference of *A*, *B* will rotate twice in one minute, while *A* only rotates once in the same minute. Therefore the RPM of *B* is twice that of *A*.

So if we use pulley *A* in Formula 4-11, we have

$$Fb = (\pi * 24 * 1)/100$$
$$= 75.4/100$$
$$= 0.754 \text{ CPM}$$

If we use pulley *B* in Formula 4-11, we have

$$Fb = (\pi * 12 * 2)/100$$
$$= 75.4/100$$
$$= 0.754 \text{ CPM}$$

If you do not know the length of a belt, and cannot remove the belt in order to measure it, you can calculate its length if you know the radius of each pulley and the distance between their centers. Figure 50 explains this process.

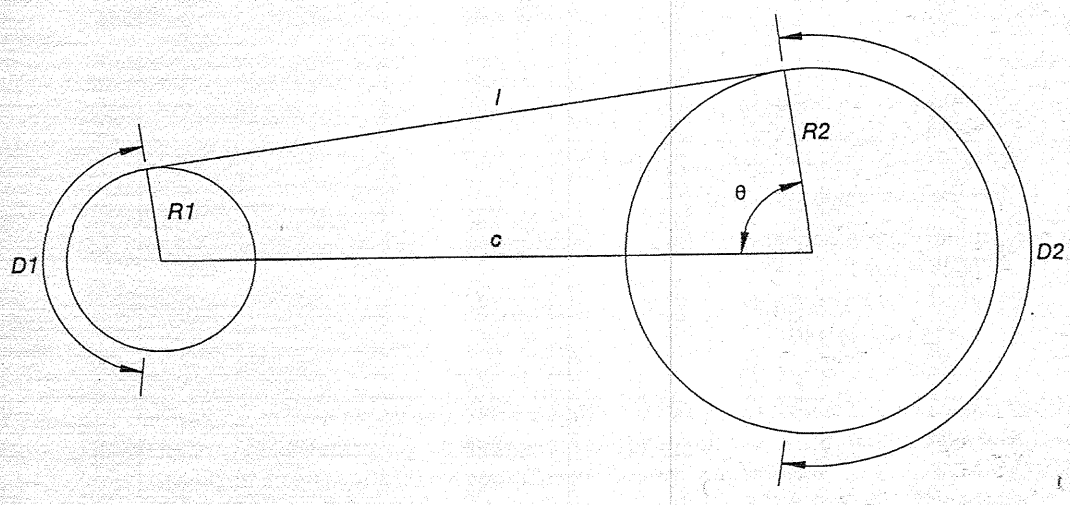


Figure 50 - *calculating belt length*

First, we find the length *l* by the formula

$$l = \sqrt{c^2 - (R2-R1)^2} \qquad \text{(Formula 4-12)}$$

where

l = belt length (in. or mm)
c = the center-to-center distance between the two pulleys (in. or mm)
R1 = the radius of one pulley (in. or mm)
R2 = the radius of the other pulley (in. or mm)

Now in order to find the length of the arcs *D1* and *D2*, we need to first calculate the angle θ. Angle θ can be found by

$$\theta = \arcsin\left(\frac{l}{c}\right) \qquad \text{(Formula 4-13)}$$

The operation "arc sin" means inverse sine, or "the angle whose sine is" (often written in math textbooks as $\sin^{-1}$). Now the arcs *D1* and *D2* can be found by

$$D1 = \pi * R1 * \left(\frac{\theta}{90}\right)$$

$$D2 = 2\pi * R2 * \left(1 - \frac{\theta}{180}\right)$$

Finally, the belt length *Bl* is equal to

$$Bl = 2 * l + D1 + D2 \qquad \text{(Formula 4-14)}$$

1.3.6 Gear Mesh Frequencies

The gear mesh frequency *GMF* is found by the formula

$$GMF = S * Nt \qquad \text{(Formula 4-15)}$$

where

GMF = gear mesh frequency (Hz)
S = frequency of the gear (Hz)
Nt = the number of teeth

As the drive gear rotates, each of its teeth impacts against the gear that it is driving as the two gears mesh. Since there is one impact per tooth, this impact occurs *Nt* times for each revolution.

For example, the gear mesh frequency of a 256-tooth gear running at 3600 RPM or 60 Hz equals

$GMF = 256 * 60$
$= 15{,}360$ Hz

When you are diagnosing a multiple-gear train, you will need to calculate the gear ratio between the drive gear and the driven gear(s). This ratio is then used to calculate the speed of the driven gears. For example, look at Figure 51. A 256-tooth drive gear spinning at 60 Hz is driving two pinion gears labeled *P1* and *P2*. *P1* has 38 teeth and *P2* has 21 teeth.

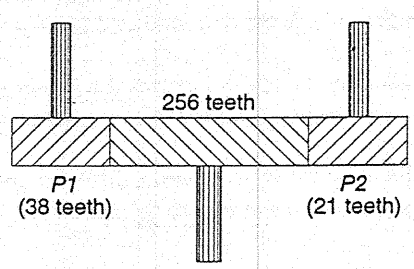


Figure 51 - *calculating gear ratios*

The gear ratio *Gr* is found by

$$Gr = \frac{\#\ teeth\ of\ driven\ gear}{\#\ teeth\ of\ drive\ gear}$$

So the gear ratios *Gr1* and *Gr2* for pinions *P1* and *P2* respectively are equal to

$Gr1 = 38/256$
$= 0.148$

$Gr2 = 21/256$
$= 0.082$

The pinion rotating speed *Rp* is found by

$$Rp = \frac{S}{Gr} \qquad \text{(Formula 4-16)}$$

where

Rp = pinion rotating speed (Hz)
S = speed of input (drive) gear (Hz)
Gr = gear ratio

The rotating speeds *RP1* and *RP2* of pinions *P1* and *P2* respectively are then

$RP1 = 60/0.148$
$= 405$ Hz

$RP2 = 60/0.082$
$= 731$ Hz

1.3.7 Planetary Gears

So far we have limited our discussion to gear box configurations involving offset parallel shafts. Another type of gear box configuration involves planetary gears. Planetary gears are used primarily for speed changes between inline input and output shafts. The input and output shafts may rotate in the same direction or in opposite directions, depending on the design of the gearbox. Planetary gearboxes may be single or double reduction. There are many different designs of planetary gears, one of which is shown in Figure 52.

In this design the input shaft ends in a flange called a **planet carrier**. The **planet** is a small gear mounted out toward the edge of the planet carrier. Surrounding the planet is a large, stationary gear called a **ring gear**. The planet engages both the ring gear and a **sun gear** which is connected to the output shaft.

As the input shaft rotates, the stationary ring gear causes the planet gears to rotate in the opposite direction against the sun gear. This causes the sun gear to rotate in the direction opposite the planets, which is the same direction as the input gear.

The overall gear ratio causes the output shaft to turn at a different speed than the input shaft. The overall gear ratio for the planetary gear shown in Figure 52 can be found by the formula

$$Gr = 1 + \frac{Nr}{Ns} \qquad \text{(Formula 4-17)}$$

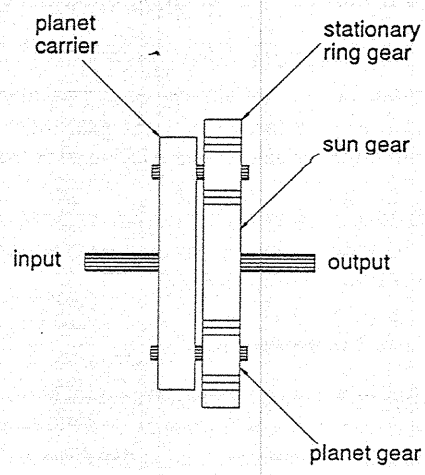


Figure 52 - *planetary gears*

where

Gr = gear ratio
Nr = number of teeth on the ring gear
Ns = number of teeth on the sun gear

The rotating speed of the planet gear in Figure 52 is found by the formula

$$Sp = Sc * \left(\frac{Nr}{Np}\right) \qquad \text{(Formula 4-18)}$$

where

Sp = speed of the planet gear
Sc = speed of the carrier (input)
Nr = number of teeth on the ring gear
Np = number of teeth on the planet gear

The gear mesh frequency of the planetary gearbox shown in Figure 52 can be found by the formula

$$GMF = Np * Sp \qquad \text{(Formula 4-19)}$$

where

GMF = gear mesh frequency
Np = number of teeth on the planet gear
Sp = speed of the planet gear

1.3.7 Blade Pass Frequencies

Blade pass frequencies occur as each blade on a rotor (such as a fan) delivers its contribution to the compressive process. As these pressure zones impact stationary parts of the machine or pass

through bends and transitions in the system, a vibration equal to the number of blades times the shaft rotation speed is created.

For example, a four-bladed fan rotates at 1,200 RPM (20 Hz). Since there are four blades, a high pressure pulse occurs four times for each revolution, or at a frequency of 4,800 CPM or 80 Hz.

This blade pass frequency is exactly the same as the gear mesh frequency discussed previously. The flexible blades on a pump impeller also create a blade pass frequency as the blades pass by the outlet port of the pump.

Blade pass frequencies can become very high if the fan, compressor, or pump is not operating close to its best efficiency point. In addition, system or duct resonances can be excited at the blade pass frequency, resulting in serious damage to the system.

1.3.8 Approximate Resonant Frequencies

There is one resonant frequency for each degree of freedom in the system (see the System Response chapter for a detailed discussion of resonant frequency and degrees of freedom). Generally the first degree of freedom causes the greatest vibration and is therefore the one of greatest interest.

One of the most common problems in industry today is the excitation of natural frequencies in the system. Most often the design has not been properly evaluated for resonances which may be excited by forcing functions that are present.

High vibration amplitudes usually indicate the presence of resonances. Beat frequencies (the sum and differences of two related frequencies) are often present when a forcing function isn't quite equal to the resonant frequency. If resonance is suspected it is important to excite the system (with a startup or coastdown examination or impact techniques) to identify system resonances and forcing functions. There are simple computer programs that can aid you in approximating shaft and rotor resonances.

1.4 Determine Where to Take Data and What Equipment to Use

Once you make a sketch of the machine and determine the possible forcing functions, you must choose the data collection locations and the transducer and instrumentation combinations best suited to solve the problem.

If the primary problem is in the low frequency area and sleeve bearings are present in the machine, you may need to use proximity probes rather than accelerometers. If you are analyzing gear boxes the reverse is true: the use of accelerometers may be necessary. See the chapter on Transducers for more information.

You must collect data in the correct plane in order to properly diagnose the vibration. There are two planes for collecting data on a rotor bearing: axial and radial. Readings in the radial plane are taken in both vertical and horizontal orientations. These planes are illustrated in Figure 53.

The **axial** plane is along the center line of the rotor; that is, the shaft's center line lies in the axial plane. Some forcing functions, such as angular misalignment or a bent shaft, have their greatest effect in the axial plane.

The **radial** plane is perpendicular to the axial plane; the radii of the shaft lie in the radial plane. Forcing functions such as imbalance, rotor rub, or loose bearings dominate the radial plane.

There are two radial plane readings that are usually taken: vertical and horizontal. A single radial reading may not be enough to properly diagnose the vibration. For example, a direct-driven fan has a very high vibration reading in the vertical direction at 2 times shaft RPM. The horizontal reading does not show the 2*RPM peak, but shows a smaller peak at 1*RPM, the operating frequency of the shaft. The 2*RPM reading could indicate misalignment, while the 1*RPM reading seems to indicate

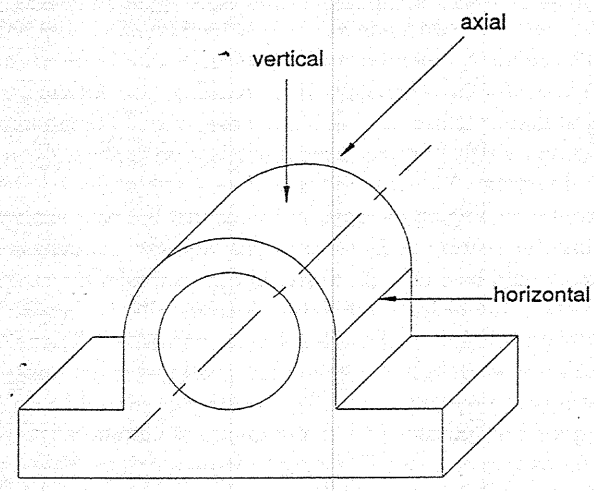


Figure 53 - *planes for collecting vibration data*

imbalance. By taking vertical, horizontal, and axial readings, you can see that the rotor is not bent, and that the 1*RPM peak was in fact caused by shaft misalignment.

If a reading had been taken only in the horizontal direction, the cause of vibration might have been mistakenly interpreted as imbalance, and a great deal of time and money would have been wasted attempting to balance a fan that did not need to be balanced.

Enough readings should be taken at each measuring point so that the rocking and translational modes of that point in space can be established. This requires several readings in the vertical, horizontal, and axial directions as shown in Figure 54. Once the motion of the point has been established, it is much easier to determine what type of forcing function could cause that kind of motion.

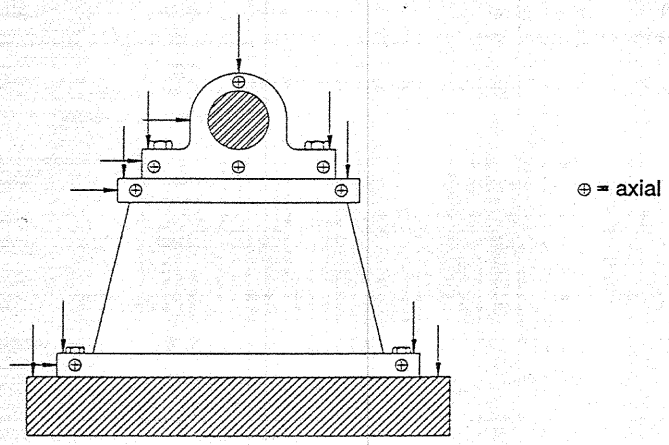


Figure 54 - *determining total bearing motion*

These readings also enable the analyst to determine the mechanical integrity of the bearing support and other bolted interfaces. Any two machine surfaces held together with bolts comprise a mechanical interface. If the bolts are tight, vibration readings taken on one side of the interface should be the same as the readings taken on the other side. If the readings are not the same, then there is a lack of mechanical integrity at that interface: the bolts are loose, one of the mounting feet is cracked, etc.

For example, suppose a single horizontal reading taken on the bearing in Figure 54 showed a high displacement in that direction. Rather than immediately assuming that imbalance is the cause, the analyst takes a horizontal reading on the base and finds that the base is also moving.

If the bolts holding the bearing pedestal are tight, and the pedestal is held firmly to the floor, movement of the base would be minimal. Since, in our example, the base is moving, this indicates that the bolts at the base of the pedestal are loose. It is very likely that the high horizontal vibration taken at the bearing would disappear if the bolts were tightened. It would be a waste of time and money to balance the shaft when tightening the bolts is all that is necessary!

Pickup placement is also very important. The transducer should be rigidly mounted on a part of the casing that has a good transmission path to the source of vibration. Bolts that hold the two halves of a bearing casing, for example, are excellent measurement points. Bearing end caps and motor end caps, however, are often not rigid and make for poor measurement points.

Finally, the vibration data should be repeatable; you should be sure that you would get the same readings if you shut the machine down and started it up again. Vibration readings that change from run to run indicate that something is changing inside the machine: rust is lying loose inside the rotor, fly ash caked on fan blades is breaking off, etc.

Hammer mills are typical of machines where the first-order vibration data changes from run to run. In balancing these units it is necessary to take several readings at each step of the balance procedure and average them so that the final scatter is centered around zero.

1.5 Take Vibration Data

Vibration data from a transducer must be converted into a meaningful form for proper diagnosis. There are two types of displays that we will consider: time domain and frequency domain. A **time domain display** shows how the amplitude of the signal varied over time. Since the amplitude of the transducer's output voltage is proportional to the amplitude of vibration, this display shows the vibration amplitude over time. Figure 55 shows a time domain display taken on a fan.

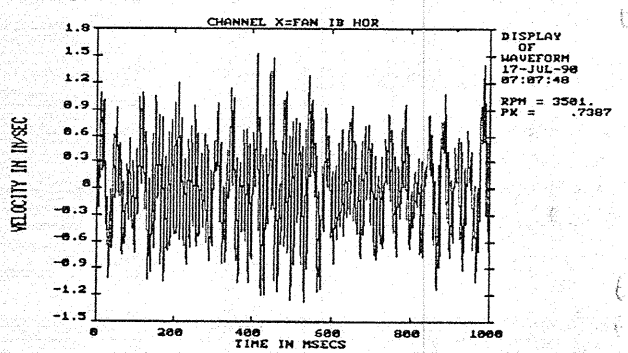


Figure 55 - *time domain data taken on a fan*

Time domain displays are typically sine waves that have undergone harmonic distortion. By using a Fast Fourier Transform (FFT) analyzer, the time domain waveform can be broken down into its pure fundamental and harmonic sine waves.

Every vibration analysis procedure should include a time domain examination of the data taken with an accelerometer to look for impacting and non-periodic events.

Time domain waveforms are useful in determining whether the vibration

- is random or periodic. Periodic signals are transformed into the frequency domain, whereas non-periodic signals are not. Non-periodic data is an indication of problems not associated with a machine component, such as arcing in electrical circuits.
- has spikes, which indicate impacts such as those associated with anti-friction bearing faults or gear tooth impacts.
- has discontinuities, asymmetries, or flattening. Intermittent problems such as electrical faults may cause the signal to come and go in an irregular fashion. Uneven loading or non-linear damping may cause the positive-going portion of the signal to be larger or smaller than the negative-going portion. One portion of the signal may be flattened, indicating a restriction of the motion in one direction.
- is electrical vs. mechanical. Time domain data can be taken during a shutdown so that the exact moment of power cutoff is included in the time window. The signal before cutoff is electrical plus mechanical and the signal after cutoff is mechanical only. This is not apparent in the frequency domain.
- changes over time with temperature or load. Long-term time domain data capture can be a powerful aid in evaluating the vibration data over time, temperature, and load.

A **frequency domain signal** is a plot of the signal amplitude on the Y axis versus frequency on the X axis. Called a **frequency signature** in the vibration industry, a frequency domain signal shows the frequencies and amplitudes of periodic components in the signal. Figure 56 shows a frequency signature from the time domain data shown in Figure 55.

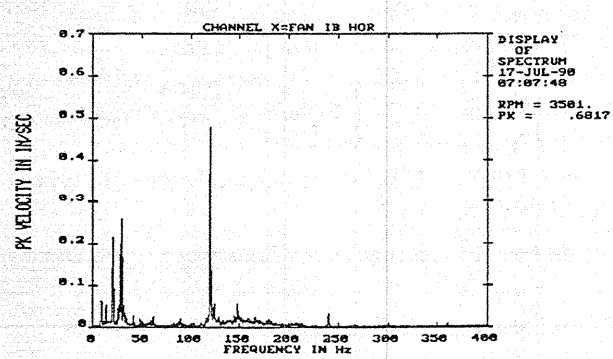


Figure 56 - *frequency domain display of time domain data from Figure 55*

Frequency domain presentations are useful in measuring

- periodic events
- the relative amplitude of a particular machine component

- harmonic relationships within the vibration signal
- the separation of very close frequencies
- the precise location of frequencies

Digital signal sampling in the time domain and transformation to the frequency domain make possible many powerful techniques for vibration analysis such as:

- instantaneous spectra
- linear or summation averaging
- exponential averaging
- peak hold averaging
- synchronous time averaging
- orders tracking
- coherence
- transfer functions
- phase

See the chapter on Digital Analysis in Volume II for more information on these analysis techniques. The frequency domain can be divided into three main areas:

1. Synchronous components
2. Sub-synchronous components
3. Non-synchronous components

1.5.1 Synchronous Components

Synchronous components (also called **phase locked** components) occur at frequencies equal to N*RPM, where N is an integer and RPM is the rotating frequency of the shaft at the position where the measurement is taken. The **fundamental frequency** (also called the **first order**) is equal to the primary rotating speed of the shaft at the location being measured, and is equal to 1*RPM.

There are a variety of possible synchronous signals within a vibration signature:

Low multiples (where N = 1 to 8) may be caused by

- **imbalance**: occurs at 1*RPM. If there is looseness within the system then there may be several harmonics of the first order.
- **pitch line runout on belt sheaves**: causes a vibration at 1*RPM and is often confused with imbalance.
- **shaft center line and bearing misalignment**: can show up at 1 or more times RPM.
- **bent shaft**: 1 or more times RPM.
- **looseness**: has many harmonics of 1*RPM frequency; second or third order harmonics often have a higher amplitude than the first order. If the problem is pure looseness, then the harmonics are mostly odd-order (3*RPM, 5*RPM, etc.).
- **blade or vane pass** (small number of blades or vanes): occurs at number of blades or vanes times RPM.

- **cavitation**: the implosion of a void on the intake blade of a fan or pump when the void reaches the pressure side. Cavitation occurs at 1*RPM with harmonics up to the number of blades. High frequency noise is also present.
- **non-linear motion**: a reciprocating or geneva motion causes vibration at multiples of the first order. In most cases the second order is the highest signal.
- **non-linear response**: harmonics of operating frequency can be caused by non-linearities in the mechanical system such as cracks or other discontinuities, non-linear damping, or non-linear spring constants.

High multiples (where N > 8) may be caused by

- **gear mesh frequencies**: occurs at the number of gear teeth * RPM. These frequencies can be very high; frequencies of 8,000 to 15,000 Hz are not unusual.
- **blade pass** (on compressors or turbines): occurs at the number of blades * RPM.
- **slot frequency on motors**: the number of rotor slots on most electric motors is greater than 30, creating a high slot pass frequency.

1.5.2 Sub-Synchronous Components

Sub-synchronous components are frequencies that are less than 1*RPM of the shaft in the bearing where the data was taken. These have many sources:

- another component within the machine.
- another machine forcing this machine to vibrate.
- **primary belt frequency**: can be calculated by using the formulas shown in Section 1.3.5. Higher harmonics may also exist.
- **oil whirl and oil whip**: occurs at approximately 40 to 48% of 1*RPM. See Section 1.3.3 for a discussion of oil whirl and oil whip.
- **rubs**: occurs very close to 1/3 or 1/2 of 1*RPM, and higher 1/3 or 1/2 order harmonics are usually present.
- **a loose anti-friction bearing in its housing**: occurs at 50% of 1*RPM, 1.5*RPM, 2.5*RPM etc.
- **FTF of ball bearings**: varies from bearing to bearing and, if present, usually indicates advanced stage of bearing failure.
- **a surge**: usually occurs from 10 to 50% of 1*RPM.

1.5.3 Non-Synchronous Components

Non-synchronous components are frequencies that are higher than 1*RPM but are not synchronous to the frequency of the shaft in the bearing where the data was taken. However, harmonic families of some forcing functions which are *not* at the rotational frequency of the shaft may also be present. A typical example is a bearing fault frequency, where first, second, and third order vibrations of an inner-race or outer-race fault are usually present.

Typical non-synchronous components include

- another component in the machine.
- another machine in the area.

- other system resonances.
- belts: in most cases the highest belt frequency is equal to the primary belt frequency times the number of pulleys. In multiple-belt systems, frequencies can occur at 2*primary belt frequency up to twice the number of belts times the primary belt frequency.
- bearing fault frequencies: since the outer race of a bearing is always under load, the outer race fault frequency is most common. Outer race frequency is approximately equal to the number of balls times FTF (the fundamental cage frequency, which is approximated by 0.4*RPM).
- electrical: electrically-caused vibrations include loose transformer laminations, the dress of conductors in a raceway, electric motors with broken or cracked rotor bars, eccentric rotors or stators, rotors loose on the motor shaft, motor rotor not magnetically centered, etc.

The background vibration levels of the machine should also be checked when the machine is shut down. In more than one case extensive analysis has tried to identify a vibration problem in a machine, when the vibration was present even when the machine was shut off!

If background vibration (caused by other machines) is repetitive, the background levels can be removed from the frequency spectrum by negative averaging.

1.6 Analyze Vibration Data

Using the phase and frequency information collected in Step 1.5, you can now make a good estimate as to the forcing function(s) present in your system. You may find quite often that more than one forcing function is contributing to the overall system vibration. If this is the case, you should always treat the most severe problem first, then tackle the next problem; always solve one problem at a time.

1.7 Make Recommendations

The following tables have been developed to help you determine the probable cause (or causes) of the vibration that you have measured. Simple suggestions are listed in the table, while more detailed information may be found in the appropriate chapters.

IF THE DOMINANT FREQUENCY IS 1xRPM IN THE AXIAL PLANE		
Phase	Probable Cause(s)	Comments
180° out	Angular Misalignment	Suspect if a 2nd order peak exists on the coupling side bearing (on either the drive or driven machine) that is greater than .25 times the 1st order AND the axial 1st order on the same bearing is greater than .1 in./sec.
	Anti-friction bearing misalignment	Suspect if 2nd or 3rd orders are as high or higher than the 1st order AND the axial 1st order on both bearings is greater than .12 in./sec.
	Bent shaft	Suspect if 1st order on both shaft bearings is greater than .2 in./sec. AND the axial vibration on both bearings is greater than .12 in./sec.
0° (in phase)	Sheave face misalignment	See **Power Transmission Components** chapter in Volume II
	Thrust bearing failure (late stages)	Suspect if harmonic groups exist and the first component of any group is greater than 3 and less than 9 times running speed AND is not an integer multiple of the run speed

IF THE DOMINANT FREQUENCY IS 1xRPM IN THE AXIAL PLANE		
Phase	**Probable Cause(s)**	**Comments**
N/A	Helical gear imbalance	See **Component Analysis: Gears** chapter in Volume II
N/A	Helical gear misalignment	See **Component Analysis: Gears** chapter in Volume II
Any angle	Overhung rotor imbalance	Suspect if 1st order peak is greater than .12 in./sec. AND the horizontal to vertical ratio is greater than .25 and less than 4 on both bearings Note: an axial vibration of less than .1 in./sec. on both bearings may also be present at run speed

OTHER DOMINANT FREQUENCIES IN THE AXIAL PLANE			
Dominant Frequency	**Phase**	**Probable Cause(s)**	**Comments**
2xRPM	180° out axially	Shaft bent at coupling	
Bearing frequency (# balls x RPM)	N/A	Thrust bearing failure (early stages)	Suspect if harmonic groups exist and the first component of any group is greater than 3 and less than 9 times running speed AND is not an integer multiple of the run speed
Gear mesh frequency (# teeth x RPM)		Transmission error in helical gears	Suspect if harmonics exist at ½ tooth frequency and other half-orders
		Pitch line runout in helical gears	Suspect if sidebands exist at (# teeth ± 1) x RPM and harmonics
		Faulty helical gear tooth	Sidebands may also be present
1xRPM and 2xRPM	Both axial and radial 180° out of phase	Combination of offset and angular misalignment	Suspect if a 2nd order exists on the coupling side bearing that is greater than .25 times the 1st order AND the axial 1st order on the same bearing is greater than .1 in./sec.

IF THE DOMINANT FREQUENCY IS 1xRPM IN THE RADIAL PLANE		
Phase	**Probable Cause(s)**	**Comments**
0° (in phase)	Eccentric or imbalanced sheave	See **Power Transmission Components** chapter in Volume II
	Static imbalance	Suspect if 1st order peak is greater than .12 in./sec. AND the horizontal to vertical ratio is greater than .25 and less than 4 on both bearings Note: an axial vibration of less than .1 in./sec. on both bearings may also be present at run speed
180°	Couple imbalance	
Any angle	Dynamic imbalance	
N/A	Spur gear imbalance	1xgear rotational frequency + GMF (gear mesh frequency) with 1xRPM sidebands
	Spur gear misalignment	High GMF + GMF harmonics (1xGMF, 2xGMF, 3xGMF)
	Bearing failure (late stages)	Check for bearing fault frequency (the bearing fault frequency causing the fault will be dominant)
Varies	Looseness	Suspect loose anti-friction bearings if ½ orders are present Suspect mechanical looseness if 1st order peak is greater than .1 in./sec. AND a harmonic group of run speed exists If you are analyzing a pump, suspect loose rotating impellers if phase varies from startup to startup
N/A	Broken rotor bar	Number of poles times slip frequency sidebands will be present around 1xRPM and 1 times line frequency, 5 times line frequency, 7 times line frequency Sidebands are **not** present when motor is unloaded ("running solo")
	Eccentric rotor	Number of poles times slip frequency sidebands will be present around 1xRPM, 1 times line frequency, 2 times line frequency, 7 times line frequency Sidebands **are** present when motor is unloaded ("running solo")
	Vertical resonance	Suspect if 1st order peak is greater than .15 in./sec. and the horizontal/vertical ratio is less than .25
	Horizontal resonance	Suspect if 1st order peak is greater than .15 in./sec. and the horizontal/vertical ratio is greater than 4

OTHER DOMINANT FREQUENCIES IN THE RADIAL PLANE			
Dominant Frequency	**Phase**	**Probable Cause(s)**	**Comments**
1 x RPM and 2xRPM	Both axial and radial 180° out of phase	Combination of offset and angular misalignment	Suspect if a 2nd order exists on the coupling side bearing that is greater than .25 times the 1st order AND the axial 1st order on the same bearing is greater than .1 in./sec.
2xRPM	180°	Offset misalignment	
		Loose sleeve bearings	Other multiples of RPM may be present
$\frac{1}{2}$ x RPM	N/A	Rotor rub	Suspect if sleeve bearings are used and ½ orders are present
Sub-harmonics of RPM	N/A	Oil whirl	Suspect if sleeve bearings are present and 1st, 2nd, or 3rd order frequency divided by run speed is less than .49 and greater than .3
	Unstable	Oil whip	Suspect oil whip if sleeve bearings are present and shaft frequency is above the second critical
	Varies	Sleeve bearing failure (early stages)	
Random broadband	N/A	Cavitation	This condition is called **starvation** on fans Harmonics of running speed and broadband noise may also be present
Bearing frequency (# balls x RPM)		Bearing failure (early stages)	Check for early presence of bearing fault frequencies
2 x belt frequency		Mismatched, worn, or stretched belts	Vibration will be inline with belt Adjustable sheaves may also vibrate at 2 x belt frequency
Gear mesh frequency (# teeth x RPM)		Transmission error in spur gears	Suspect if harmonics exist at ½ tooth frequency and other half-orders
		Pitch line runout in spur gears	Suspect if sidebands exist at (# teeth ± 1) x RPM and harmonics
		Faulty tooth in spur gear	Sidebands may also be present
# blades x RPM		Blade/vane pass	
2x line frequency		Loose windings	Sidebands will be present around 2 x line frequency and around slot frequency
		Loose iron	

OTHER DOMINANT FREQUENCIES IN THE RADIAL PLANE			
Dominant Frequency	Phase	Probable Cause(s)	Comments
2 x line frequency	N/A	Stator problems	
		Loose stator	
		Phase imbalance	
		Stator out of round	

SUMMARY

Vibration problems are solved by identifying the forcing function(s) and eliminating or minimizing their effect. A general step-by-step procedure may be followed during analysis of a vibration problem.

The analysis of a vibration problem can be broken into seven steps:

1. Identify the problem.
2. Gather information about the vibrating system.
3. Determine the possible forcing function(s).
4. Determine where to take data and what equipment to use.
5. Take vibration data.
6. Analyze vibration data.
7. Make recommendations.

MACHINE INFORMATION DATA SHEET

MACHINE NAME: _____ MACHINE ID CODE: _____

MACHINE LOCATION: _____

DRIVER/MOTOR INFORMATION

DRIVER TYPE: _____ BRAND: _____ RPM: _____ HP: _____

MANUFACTURER ID#: _____ CONSTANT SPEED? _____ FRAME #: _____

BEARINGS — OUTBOARD: _____ INBOARD: _____

SLEEVE OR ROLLING ELEMENT? (CIRCLE ONE ⇒) SLEEVE ROLLING ELEMENT

IF DIRECT COUPLED, TYPE OF COUPLING: _____

IF BELT DRIVEN, # OF BELTS: BELT TYPE/#: _____ # OF SHEAVES: _____

DIAMETER OF SHEAVE #1: _____ DIAMETER OF SHEAVE #2: _____

BELT LENGTH: _____ OR CENTER-TO-CENTER DISTANCE OF SHEAVES: _____

GEARBOX? _____ IF YES, MANUFACTURER: _____ I/O RATIO: _____

SHAFT	# OF TEETH	BEARING ID#
INPUT	_____	_____
1ST INTERMEDIATE	_____	_____
2ND INTERMEDIATE	_____	_____
3RD INTERMEDIATE	_____	_____
OUTPUT	_____	_____

DRIVEN INFORMATION

DRIVEN TYPE (FAN, PUMP, GENERATOR, ETC.) _____

MANUFACTURER: _____ MANF. ID#: _____ RPM: _____

CIRCLE ONE ⇒ STANDBY INTERMITTENT CONTINUOUS

IF FAN OR PUMP, # OF VANES/BLADES (⇐ CIRCLE ONE) _____

BEARING TYPE (CIRCLE ONE ⇒) SLEEVE ROLLING ELEMENT

BEARING #1 ID: _____ BEARING #2 ID: _____

COMMENTS:

5 PHASE

There are many situations where it is very important to know the true phase relationships that exist in a vibrating system. Probably the most common phase measurements made today are relative. That is, phase shifts which may exist in the transducer or the vibration instrumentation are assumed to be constant, and the phase measured is not the actual phase between the excitation and the response but is only "related" to it. In fact, if phase shifts are present in the measurement system, they are never constant over the frequency range. For modal and structural work it may be important to know the precise phase lag between the forcing function and the response.

To illustrate the concept of true phase measurement, we will use an unbalanced rotor as a system with a forcing function due to the imbalance and a response in the structure to that unbalance force.

Among other things, knowledge of true phase relationships makes it possible to field balance a rotor in the least number of runs; many times this can be done in only one run. To do this, you must fully understand phase measuring techniques.

1.0 MEASURING PHASE

There are two primary ways to measure phase:

1. A strobe light fired by the vibration signal
2. A readout device in conjunction with a reference signal

The readout device may be a meter, a computer screen, a spectrum analyzer, or any other means of displaying the phase reading.

In the System Response chapter, you saw that the following parameters are present simultaneously in a system vibrating in equilibrium:

1. Amplitude
2. Velocity
3. Acceleration
4. Frequency
5. Phase

The first three of these are amount parameters, and the last two are time parameters. Industry normally prefers displacement (double amplitude) rather than amplitude to indicate the peak-to-peak distance of the motion (the space occupied by the motion) in a vibrating system.

Each of the three most common types of vibration transducers produce an output voltage which is proportional to one of the three amount parameters: displacement, velocity, and acceleration. Since both methods of measuring phase employ a vibration transducer, it is important to understand the transducer characteristics.

1.1 Transducer Characteristics

In the System Response chapter, you saw that velocity leads displacement by 90 degrees and that acceleration leads velocity by 90 degrees and displacement by 180 degrees. Therefore

- the voltage output of a displacement transducer should be in phase with the motion
- the voltage output of a velocity transducer should lead the motion by 90 degrees
- the voltage output of an accelerometer should lead the motion by 180 degrees

In some cases (such as older seismic velocity transducers) the output leads have been reversed in the connector so that the output of the transducer lags (rather than leads) the displacement signal.

Velocity Transducers

The seismic velocity transducer is a typical highly-damped spring-mass system and has its own phase lag in the region of its resonant frequency. Normally the transducer phase lag occurs in the 400 to 2000 CPM frequency range and should be determined for the particular transducer being used.

Hybrid velocity transducers such as accelerometers with built-in integration circuitry do, or at least should, put out a signal that leads the displacement by 90 degrees. The integration is rolled off at some low frequency (called the **knee** of the integration) since integration at zero frequency would result in an infinite value. This is usually done at a low frequency, around 2 Hz or lower.

These hybrid velocity transducers also have a phase lag in the region of the integration knee, but this normally occurs at a much lower frequency than that of seismic-type velocity transducers. You must determine the phase versus frequency characteristics of these transducers if you want to measure true phase.

Accelerometers

The maximum working frequency for an accelerometer is usually specified as 10% of its resonant frequency; therefore, accelerometers operate far below their natural frequency. Since accelerometers operate far below their first resonant frequency, they are known as "true phase" transducers: that is, there is no phase shift within the transducer itself over its normal operating frequency range.

You can measure the transducer and the transducer/instrument phase shifts, if any, with a phase meter or an oscilloscope using a shaker table with a standard output.

1.1.1 Measuring Transducer/Instrument Phase Shifts

If this equipment is not available, you can check the phase characteristics of the instrumentation over a limited frequency range by suspending a small double-ended motor (such as a sewing machine motor) as a pendulum. Figure 57 illustrates this procedure.

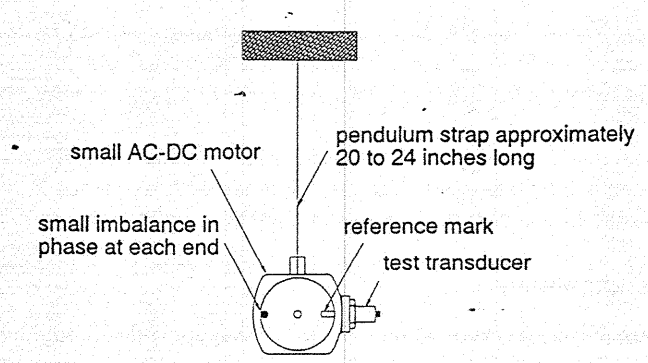


Figure 57 - *measuring transducer phase shift*

Discs are mounted on both shafts and the motor is precisely balanced. Then, a small weight is added in phase (i.e. at the same angular location) to each disc. The pendulum system has a very low frequency, in the order of .2 to .8 Hz. Over the frequency range of the motor, say from 10 Hz to 200 Hz, the phase between the unbalance force and the motion is 180 degrees (since the system is operating well above its resonant frequency).

Mounting the test transducer as shown in Figure 57 and using a photo reference transducer makes it a simple matter to check the phase of the instrument, the transducer, or the transducer/instrument combination against 180 degrees.

2.0 SYSTEM RESPONSE AND PHASE

It was shown in Section 12 of the System Response chapter that all mechanical systems are operating close to a damped resonant frequency because of competition, wear, and customer demands. The typical system response and phase curves for such a systems are shown in Figure 58.

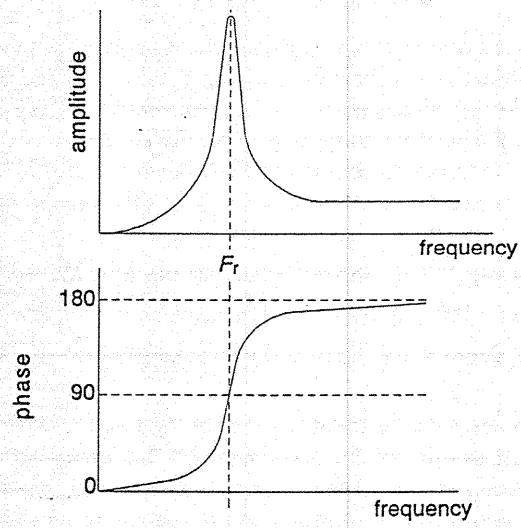


Figure 58 - *system response and phase characteristics*

The phase curve shows that the direction of the movement caused by the forcing function lags behind the direction of the forcing function somewhere between 0 and 180 degrees. This lag depends on the relationship between the operating frequency, the damping, and the resonant frequency of the system.

Since most systems in industry run below the first resonance, the phase lag for these systems is somewhere between 0 and 90 degrees. The change in lag with frequency is very rapid in the resonant region and is affected by the damping, except at resonance where it is always exactly 90 degrees.

2.1 Finding the Imbalance

From the system response curves you can see that if you know the direction of rotation, the direction of measurement, and the transducer/instrument phase characteristics, you can determine the hemisphere in which the imbalance exists. For example, assume that the direction of rotation of a shaft is clockwise, the transducer is located at the 3 o'clock position, and the transducer/instrument combination has no internal phase shifts.

Depending on the shaft's operating frequency in relation to the resonant frequency the imbalance phase reading is between 3 o'clock (a lag of 0 degrees) and 9 o'clock (180 degrees). The reading would be exactly 6 o'clock if the system were running at the resonant frequency (a lag of 90 degrees).

If you know both the operating frequency and the resonant frequency of the shaft, you can now determine the quadrant in which the imbalance exists. For example, if the operating frequency is below the resonant frequency, the imbalance is between 3 o'clock (a lag of 0 degrees) and 6 o'clock (a lag of 90 degrees). If the operating frequency is above the resonant frequency the imbalance is between 6 o'clock (90 degrees) and 9 o'clock (180 degrees).

If you know the actual phase lag of the system, and the phase measuring characteristics of the instrumentation, then you can determine the actual location of the imbalance. Finally, if you know the response of the system to a known force as well as the phase lag, then you can calculate both the magnitude of the imbalance and its location.

2.2 Measuring Phase with a Strobe Light

Analog analyzers which use a strobe light to indicate phase usually split the signal input from the transducer. One portion is filtered, scaled, rectified, averaged, and displayed on an analog or a digital meter as peak amplitude, peak-to-peak displacement, or peak velocity. The other portion is "squared up" and differentiated so that the positive-going pulse fires the strobe.

If there were no phase shifts in the electronics of the analyzer, and if a "true phase" transducer were used to measure the vibration, the strobe would fire from this squared-up signal, depending on the type of transducer:

- **Displacement Transducer** - 90 degrees before the machine moved toward the transducer
- **Velocity Transducer** - 180 degrees before the machine moved toward the transducer
- **Accelerometer** - 90 degrees after the machine moved toward the transducer (that is, 270 degrees before)

As discussed previously, some velocity transducers have the leads reversed in the connector, and all velocity transducers have phase shifts of their own in the low frequency region. In addition, almost all analog analyzers have some phase shift in their electronics. It is very important to know the phase characteristics of the transducer/instrument combination if you are going to measure the magnitude and location of the imbalance with this type of equipment.

2.3 Measuring Phase with a Reference Signal

When phase is measured using a reference signal, as shown in Figure 59, the angle can be measured from the reference signal to the vibration signal or from the vibration signal to the reference signal. The angle *R* from the reference to the vibration is equal to -1 times the angle *V* from the vibration to the reference (*R = -1 * V*). This formula also shows that the angle *V* is equal to -1 times the angle *R*.

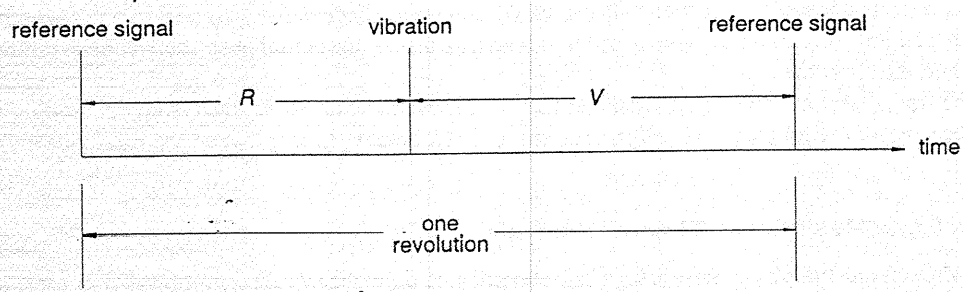


Figure 59 - *system phase response and measurement*

One revolution is equal to 360 degrees. If R were equal to 30 degrees then *V* would be equal to 330 degrees (since 330 degrees is equal to -30 degrees). This relationship is illustrated in Figure 60.

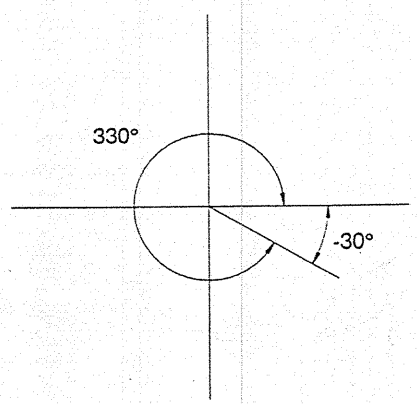


Figure 60 - *positive and negative angles*

3.0 CROSS CHANNEL TECHNIQUES

Most present day analyzers measure phase from the reference signal to the vibration signal. Two-channel spectrum analyzers with cross channel capability (usually channels A and B, sometimes channels X and Y) almost always measure the phase from channel B (or Y) to channel A (or X). You must know the phase measuring method of the equipment being used if you are measuring true phase.

Although some instruments which use a reference signal to measure phase have phase shifts in the electronics, the cross channel phase measured with a two-channel spectrum analyzer has no instrument error in the phase reading.

If a reference signal with no phase shift in the reference signal circuitry is input to one channel of a two-channel digital analyzer and the output from a true phase transducer is input to the other channel, then both the magnitude and the phase of the imbalance vector are measured. The phase is either the R or the V phase shown in Figure 59, depending on the channels chosen for the reference signal and the vibration signal. If V is measured then R = -1 * V.

This measured phase is the phase between the reference signal and the transducer signal. If the transducer is an accelerometer then, since acceleration leads displacement by 180 degrees, you must subtract 180 degrees from the value in order to read true phase (the actual phase between the reference signal and the motion).

If the transducer is a velocity transducer then you must subtract 90 degrees from the reading plus the phase lag of the transducer itself in order to read the actual phase between the reference and the motion.

3.1 An Example of Cross Channel Analysis

To understand the phase measuring characteristics of transducer/instrument configurations, consider the unbalanced rotor shown in Figure 61. The imbalance generates a centrifugal force F(t) which is a rotating vector whose magnitude is radially out from the center of rotation and whose phase is constantly changing.

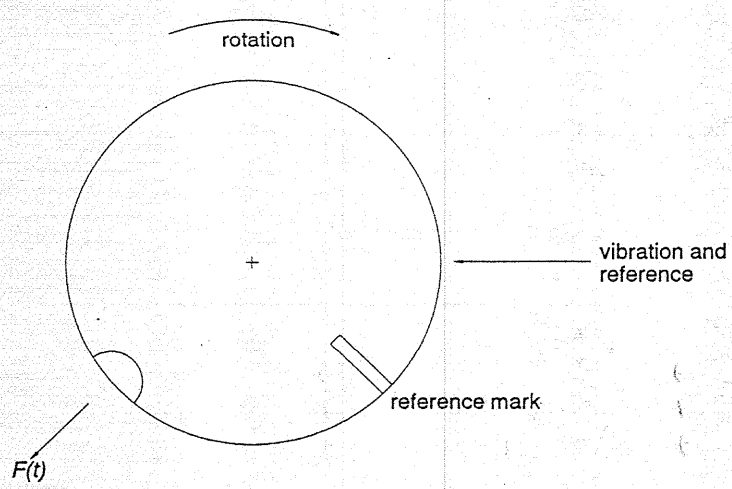


Figure 61 - *an unbalanced rotor*

The direction of rotation is clockwise. An accelerometer is mounted as shown and input into channel B (or Y) of a two-channel spectrum analyzer. The output of a no-phase shift reference transducer is input into channel A (or X), using a Hanning window to minimize leakage (Hanning windows are discussed in the Digital Analysis chapter in Volume II).

The rotor is run at operating frequency and, using the analyzer's cross channel capabilities, both the amplitude and phase of the unbalance are read as

.98 g's @ -247 degrees

Since almost all spectrum analyzers read phase from channel B (or Y) to channel A (or X), the real angle between the reference signal and the acceleration is not -247 but +247 degrees (remember, we want the angle *R*, not the angle *V*). Since the transducer is an accelerometer, we must subtract 180 degrees so that the true phase between the reference signal and the motion is 247 - 180 = 67 degrees. As pointed out before, the true phase of the imbalance vector is between 0 and 180 degrees.

The actual location of the imbalance in the rotor is given by the equation

$$H = K - R - T \qquad \text{(Formula 5-1)}$$

where

H = degrees from the reference mark to the actual heavy spot in the direction of rotation

K = the phase lag of the mechanical system

R = the true phase measured by the spectrum analyzer (in this example, 67 degrees)

T = the angle in the direction of rotation between the vibration transducer and the reference transducer (in this example, 0)

USE FORMULA 5-1 CORRECTLY

Formula 5-1 assumes that angles increase in the direction of rotation, since that is the way the transducer reads the heavy spot. First the reference transducer reads its mark. Then, after some angular separation, the heavy spot sweeps by the vibration transducer. This angular separation is positive, since it is a measure of the time between the two events.

However, a popular industry convention numbers angles increasing *opposite* to the direction of rotation. This is exactly opposite to how the transducer sees the heavy spot. If you are numbering angles in this manner, then Formula 5-1 will not provide the correct answer.

When numbering angles to increase opposite to the direction of rotation, use the formula

$$H=R-K+T$$

to determine the location of the imbalance.

If you know the system phase lag *K*, you can determine the location of the imbalance. If you know the system response to a known force, you can determine both the magnitude and location of the imbalance.

4.0 OTHER CROSS CHANNEL CAPABILITIES

The other cross channel capabilities of a two-channel spectrum analyzer can be used to determine both the system phase lag and the system response to a known force.

Three important cross channel parameters are coherence, phase, and the transfer function. Coherence is a measure of the linear relationship between two signals, the transfer function is the ratio of two linearly related signals, and phase is the phase angle between the signals. See the chapter on Digital Analysis in Volume II for a more detailed discussion of these parameters.

4.1 Transfer Functions

There are three usual transfer functions:

1. Inertance = Acceleration/Force (g's/lb)
2. Mobility= Velocity/Force (in/sec/lb)
3. Compliance = Displacement/Force (mils/lb)

and their reciprocals:

1. Mass = Force/Acceleration (lb/g's)
2. Impedance = Force/Velocity (lb/in/sec)
3. Stiffness = Force/Displacement (lb/mils)

If the output from an instrumented force hammer is input into channel A (or X), and the output from the accelerometer in our example is input into channel B (or Y) of the two-channel spectrum analyzer, it is possible to impact the mechanical system and determine the system's phase lag and response.

It is best to impact the rotor in the same direction as the transducer at the correction plane locations and measure the response at the same location from which the operating data was taken. If it is not possible to impact the rotor at this point, the bearing is impacted in the direction of the transducer as shown in Figure 62.

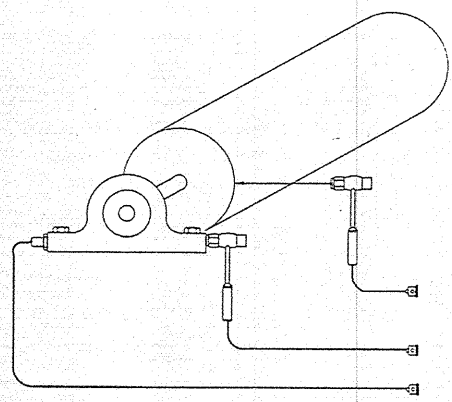


Figure 62 - *impact directions*

Since the coherence is 100% on the first average, it is important to take enough averages in order to obtain a valid coherence. If the coherence is better than 85-90% between the force hammer and the response the transfer function data may also be considered valid. If after 3 or 4 averages the coherence is less than 80%, there are probably structural non-linearities that need to be investigated.

If there is good coherence between the signals, then the transfer function can be used to determine the system characteristics. Since the transfer function is usually channel B/A (or Y/X), and since the force hammer was input to channel A and the accelerometer was input to channel B, the measured transfer function is inertance (g's/lb), and the phase is the phase between the acceleration response and the input force.

4.1.1 Using the Transfer Function

To continue with the example from Section 3.1, use the instrumented force hammer and impact the system at several locations, taking 8 averages and checking the coherence. If the coherence is

better than 90% then the transfer function and phase can be read from the analyzer. Suppose our reading is:

Inertance = .25 g's/lbs @ 18 degrees

Since the phase is between channel B and A and the desired phase is between the force and the response, once again the phase reading is not +18 but -18 degrees. Therefore the system phase lag between the force and the motion (displacement) is 180 - 18 = 162 degrees. Remember, the 180 degrees is due to the fact that acceleration leads the motion by this amount. Now the actual location of the unbalance in the example can be calculated from Formula 5-1

$$H = K - R - T$$

since the angle between the transducer and the reference in the example was set to zero, $T = 0$. The operating phase was measured as $R = 67$, and the system phase lag has been estimated to be 162, so

$$H = 162 - 67 - 0 = 95 \text{ degrees}$$

That is, the heavy spot is 95 degrees in the direction of rotation from the reference mark.

Since the system response was measured as .25 g's per pound of force, it is also possible to calculate the magnitude of the unbalance from the formula

$$U = \frac{X}{I} \qquad \text{(Formula 5-2)}$$

where

U = Unbalance (lb.-in.)
X = Operating displacement (inches Peak-Peak)
I = Inertance (g's per lb.)

If the operating displacement is measured in mils peak-to-peak and the calculated unbalance is desired in ounce-inches, we must multiply the pounds by 16 to get ounces and divide the mils peak-peak by 2,000 to get inches. The equation then becomes

$$U = .008 * \frac{X}{I}$$

In our example the vibration was measured in g's peak rather than mils peak-to-peak, so the unbalance is given by

$$U = \frac{156.6 * g}{I * f^2} \qquad \text{(Formula 5-3)}$$

where

U = Unbalance (oz.-in.)
g = Operating vibration (g's Peak)
I = Inertance (g's/lb.)
f = Operating frequency (Hz)

that is, the actual unbalance in the rotor equals

$$U = \frac{(156.6 * .98)}{(.25 * 900)}$$
$$= .68 \text{ oz-in}$$

located 95 degrees in the direction of rotation from the reference mark.

If you have taken care while taking the data and measuring true phase, a correction of .68 oz-in added 180 degrees from this heavy spot should balance the rotor without the necessity of adding trial weights, calculating the correction amount and angle, and making additional runs.

Although we have used an unbalanced rotor to illustrate the concepts of true phase measurements, the same precautions and techniques should be used where phase shifts with frequency in the transducers or the instrumentation may be present. Situations where the true phase between machine components or the structure is vital to the solution of the vibration problem also require care.

SUMMARY

There are many situations where you need to know the true phase relationships that exist in a vibrating system. Knowledge of phase relationships makes it possible to field balance a rotor in the least number of runs; many times this can be done in only one run.

There are two primary ways to measure phase:

1. A strobe light fired by the vibration signal
2. A readout device in conjunction with a reference signal

Since both methods of measuring phase employ a vibration transducer, it is important to understand the transducer characteristics.

The voltage output of a velocity transducer should lead the motion by 90 degrees and the voltage output of an accelerometer should lead the motion by 180 degrees. In some cases, the output leads have been reversed in the connector so that the output of the transducer lags (rather than leads) the displacement signal.

The seismic velocity transducer is a typical highly-damped spring-mass system and, as such, also has its own phase lag in the region of its resonant frequency. Normally this occurs in the 400 to 2000 CPM frequency range and should be determined for the particular transducer being used.

Hybrid velocity transducers such as accelerometers with built-in integration circuitry put out a signal that leads the displacement by 90 degrees. Since accelerometers operate far below their first resonant frequency, they are "true phase" transducers: that is, there is no phase shift within the transducer itself over its normal operating frequency range.

The direction of the movement caused by the forcing function lags behind the direction of the forcing function somewhere between 0 and 180 degrees. This lag depends on the relationship between the operating frequency and the resonant frequency of the system. Since most systems in industry run below the first resonance, the phase lag for these systems is somewhere between 0 and 90 degrees.

You can determine the hemisphere in which the imbalance exists if you know the direction of rotation, the direction of measurement, and the transducer/instrument characteristics. If you know the operating frequency and the resonant frequency, then you can determine the quadrant in which the imbalance exists.

If you know the actual phase lag of the system and the phase measuring characteristics of the instrumentation, then you can determine the actual location of the imbalance. If you know the response of the system to a known force as well as the phase lag then you can determine both the magnitude of the imbalance and its location.

6 FIELD BALANCING

The balancing of rotating assemblies in the field is not only an efficient, but many times a necessary, procedure in order to obtain an acceptable running balance. Rotors balanced in a production balancing machine (i.e. before the machine is assembled) are not:

- balanced in their own bearings
- balanced at the normal running frequency
- balanced at normal load condition
- driven as they are designed to be driven

Thus the operating frequency, the rotor stiffness, the bearing support and its stiffness, and the load and drive conditions all contribute to a final running condition of the assembly which may not meet the required vibration specification.

Once put into service, the rotor balance may change due to stress relieving, erosion, buildup, etc., and as the first-order vibration increases it becomes necessary to correct the rotating assembly balance.

1.0 ADVANTAGES OF FIELD BALANCING

Balancing such assemblies in the field (and in place) has several significant advantages:

1. The rotor is balanced in its own bearings.
2. The rotor is balanced at the normal operating frequency.
3. The rotor is balanced at normal load.
4. The rotor is driven as it is driven during normal operation.
5. Tear down, re-assembly, and re-alignment are not necessary.
6. A final in-place trim balance is not required.
7. Down time is greatly reduced.

Because of the practical and financial advantages of correcting installed rotating assemblies without teardown, and since computers, calculators, and the vibration instrumentation required to take the data are readily available, it is important to understand the basic principles of the in-place balancing procedures.

These are not difficult procedures. The greatest problems in most field balancing operations are

- starting and stopping the machine.
- adding or removing correction weight to or from the rotor.

2.0 PRELIMINARIES TO FIELD BALANCING

Since it is an industry-wide fallacy that imbalance is the most common cause of machinery vibration, there is a tendency to do inadequate, or no, vibration analysis before attempting a balance operation. If the field balance procedure does not work, it is because the vibration analysis was not carefully done or the rotor dynamics were not understood.

Never attempt to perform field balancing until you have determined that the forcing function is rotor imbalance. In addition, investigate the rotor resonance, bearing support resonance, and other systems resonances to insure that the operating frequency is at least 20 (better 30) percent above or below these resonances.

You must understand the fundamentals of system response before discussing field balancing procedures. Please refer to the System Response chapter for a complete explanation of system response.

Once you have determined through careful analysis that imbalance is the forcing function and that a field balancing procedure should be performed, your initial preparations should include

1. **Transducers mounted at each bearing.** The transducers should be rigidly attached to the bearing or the bearing support, never hand-held, and should not be moved during the entire procedure.

2. **Provision to read phase within 5 degrees.** Take phase readings from one position throughout the balance operation, even if there is more than one correction plane. If you cannot read phase within 5 degrees it may take more runs than normal to achieve an acceptable vibration level. This increases downtime and the stress on the machine due to starting in-rush (the sudden addition of a mechanical or electrical load). Many machines are also limited in the number of starts in a given time period.

3. **The ability to maintain the same operating frequency** for each run during one set of balance test runs.

4. **Confidence that the vibration data is repeatable.** If the data from a balance run cannot be repeated, it may be due to loose parts, chips, water soluble lubricant, rust, water, and other types of foreign material in the rotor.

It is possible that the vibration is not due to imbalance. There are rotors such as hammermills where the vibration readings do not repeat from one run to the next. The imbalance in this type of rotor is due to the hammers not assuming the same relative position each run. This causes the imbalance readings to "scatter" around a single point that represents the average imbalance. In order to minimize the vibration in these units, it is necessary to take readings over several runs at each step of the balance procedure. Then you average the readings at each step so that, at the conclusion of the

balance procedure, the vibration scatter is centered around zero. Since the location of such matter can change from one run to the next, it is impossible to calculate accurate balance correction weights.

You can measure phase with a strobe light utilizing either:

- a reference mark on the rotating element and reading angles from the stationary element
- a reference mark on the stationary member and reading phase angles from the rotating member

You can also measure phase using a reference transducer, such as a photo tack or a magnetic proximity pickup, displaying the result on a readout device such as a meter, a spectrum analyzer, or an oscilloscope. See the chapter on Phase for a more detailed discussion on measuring phase with digital instruments and using "true phase" to minimize the number of balance runs.

3.0 SINGLE PLANE BALANCE

We can illustrate the basic techniques of field balancing by looking at imbalance and imbalance correction in one plane, often called **single plane** (or **static**) balance.

Single plane balance is a dynamic or running balance conducted in only one correction plane. In actual practice, you should add balance correction weights in as many correction planes as necessary in order to obtain an acceptable running condition. In some cases this may only be one plane.

Perfect balance exists when the running center and the mass center of the rotor coincide. When an imbalance condition develops, shown in Figure 63 as a mass concentration *Ub* at some location, the centrifugal force of the imbalance mass is

$$Fc = \omega^2 * \frac{Ub}{g} \qquad \text{(Formula 6-1)}$$

where $Ub = M * r$, and

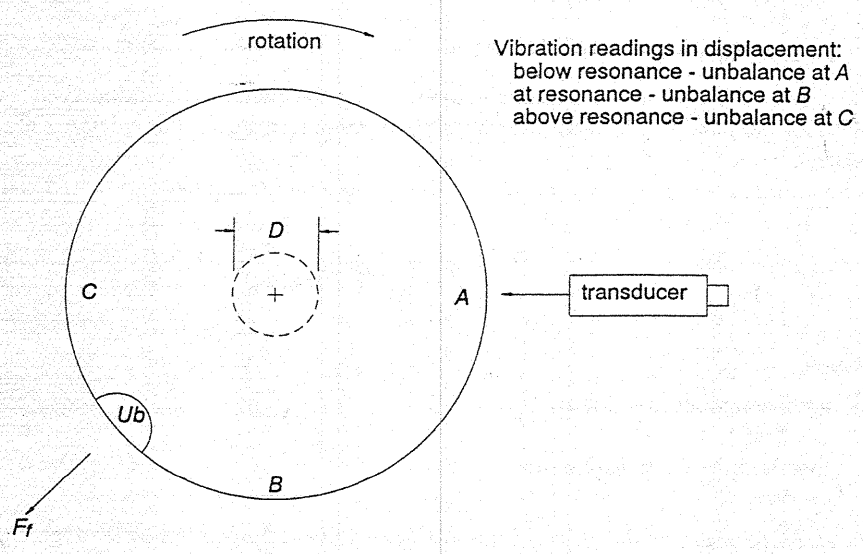


Figure 63 - *an imbalance condition*

Fc = centrifugal force
Ub =rotor imbalance (oz.-in.)
M = weight of the imbalance(oz.)
r = radius from rotor center to M (in.)
$\omega = 2\,\pi f$
f = frequency (Hz)
g = acceleration due to gravity (ft./sec.)

This radial force is the system forcing function $F(t)$, and acting against the bearing support stiffness results in an orbital motion of this support at the same frequency as that of the rotor.

This motion, vibration, is measured by attaching a vibration transducer at the bearing. Even though you can measure higher amplitude signals at some other location, mount the pickup to the bearing. The bearing is not only the reaction point to ground, but it is important to minimize bearing movement so as to reduce the stress on the lubricant film in the bearing.

The vibration transducer is designed so that it is sensitive to motion along its axis and is insensitive to motion at 90 degrees to this axis. Therefore the vibration measured is proportional to the motion along the axis of the transducer. Even though the bearing motion is orbital, the vibration is measured as translational along the axis of the vibration pickup.

Assuming that the mechanical system is linear, this motion is sinusoidal, and the space displaced by the motion is measured as the peak-to-peak displacement in mils (.001 inch), or in mm. If this total space is D, then the motion can be represented mathematically as

$$d = \frac{D}{2} * \sin(\omega t) \qquad \text{(Formula 6-2)}$$

where

d = instantaneous amplitude (mils)
D = Peak-to-Peak displacement (mils)
$\omega = 2\,\pi f$
t = time (seconds)

The output voltage of the transducer used during the balance operation is proportional to the vibration motion, and if this transducer is a displacement transducer, the voltage is also in phase with the motion.

If the transducer is a velocity transducer, the output voltage can be represented mathematically as

$$v = \frac{dd}{dt} = \omega * \frac{D}{2} * \cos(\omega t) \qquad \text{(Formula 6-3)}$$

where

v = instantaneous velocity
$\omega = 2\,\pi f$
D = Peak-to-Peak displacement
t = time

The cosine indicates that the phase of this voltage will lead the motion by 90 degrees.

If the transducer is an accelerometer, the output voltage is

$$a = \frac{dv}{dt} = -\omega^2 * \frac{D}{2} * \sin(\omega t) \qquad \text{(Formula 6-4)}$$

where

a = instantaneous acceleration
$\omega = 2\pi f$
D = Peak-to-Peak displacement
t = time

and this voltage will lead the motion by 180 degrees.

Although velocity and acceleration are normally the best parameters to use for the vibration analysis of a mechanical system, using displacement for the balance operation insures uniformity in the phase measurements. Displacement also simplifies the calculation of trial and correction weights.

You must understand the phase characteristics of the transducer and the vibration instrumentation employed in the balance operation so that you can determine the approximate location of the imbalance. Figure 63 illustrates one such arrangement, and shows that the imbalance force will always be in only one hemisphere. If you know the approximate operating point on the system response curve, you can estimate the imbalance location to within 30 or 40 degrees. See the chapter on Phase for more information on estimating the imbalance location.

3.1 The Single Plane Balance Procedure

The single plane balance procedure consists of four steps:

1. The "as is" run
2. The trial weight run
3. Calculate and make permanent correction
4. The final run

3.1.1 The As Is Run

The "as is" run is very important, for it provides the baseline data for all future calculations. If there is any doubt as to the repeatability of the data, the run should be repeated until it is possible to either determine that the rotor cannot be balanced or to pick an average center to any scatter in the data. Use this center as the initial data, so that after balance correction the scatter will be about zero.

The initial data consists of a vibration amplitude and a phase angle determined by the transducer and instrumentation in use. In many cases there is a beat frequency caused by other forcing functions close to the operating frequency of the rotor, such as a belt drive or another rotor. Beat frequencies are frequencies that result from the sum and difference of two related frequencies. If synchronous time averaging or vector averaging is not available, then read the average of the beat in amplitude and phase.

Assume an initial reading of

A = 10 @ 30

The magnitude of 10 mils is the peak-to-peak vibration displacement caused by the imbalance, and the phase of 30 degrees is a relative phase with respect to some reference mark. The desired information, on the other hand, is the amount of the imbalance in units of weight and the actual location of the imbalance on the rotor.

Since imbalance is always a mass at some radius the units are in ounce-inches, gram-centimeters, etc., but not mils. Imbalance is also at some location on the rotor, not at a relative phase (for example, 24 degrees from the reference transducer). We need known quantities in order to solve the equation.

If you add a trial weight of known weight at a known distance from the center of rotation, and at a known location, then you can easily solve the balance equation.

3.1.2 The Trial Weight Run

Each bearing and bearing support has several degrees of freedom. The degree of freedom whose frequency is closest to the operating frequency of the rotor is usually of greatest importance to the field balance procedure. Typical mechanical response at any degree of freedom is illustrated in Figure 64.

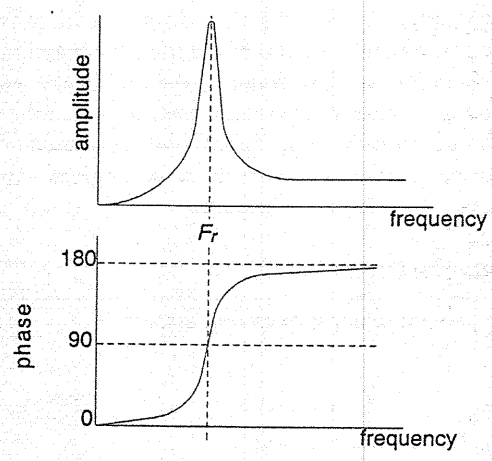


Figure 64 - *mechanical response at a degree of freedom*

You must shut the system down in order to add the trial weight. If you carefully observe the amplitude and phase of vibration immediately after shut down, you can predict the approximate running point on the response and phase curves.

For example, if the phase does not change dramatically immediately after shutdown, the operating point is not close to the system resonance. On the other hand, if the phase changes dramatically immediately after shutdown, and the amplitude rises and falls, the operating point is just above the system resonance. If the amplitude only falls, the operating point is just below the system resonance. These observations must be made immediately after shutdown, not during coast down.

Since the phase lag goes from 0 to almost 180 degrees as the operating frequency passes through the system resonance, you can now predict the approximate system phase lag. When you add the approximate phase lag to the phase characteristics of the transducer and the vibration instrumentation being used, you determine the approximate location of the rotor imbalance. The approximate location of the operating frequency on the system response curve gives an indication of the amplification factor (i.e. how much the system amplifies the vibration).

The approximate trial weight can be calculated once the rotor weight is known. However, the weight of the rotor is only part of the total weight that is vibrating. The rotor is mounted in bearings,

and the bearings in turn are mounted on pedestals. So the weights of each must be added to the rotor weight to determine the total vibrating weight.

However, the entire pedestal is not vibrating; that portion that is fastened to the foundation vibrates very little. A good estimate of total vibrating weight is to take 10% of the rotor weight and then add that number to the rotor weight.

Once you have calculated (or estimated) the vibrating weight, you can now calculate the approximate trial weight to be added for the trial weight run. The best trial weight to add is one which results in the same vibration amplitude measured in the as is run, and is 180 degrees opposite from the imbalance location.

Neglecting any amplification due to resonance, the amount of imbalance is found by the formula

$$Ub = w * Xf \qquad \text{(Formula 6-5)}$$

where

Ub = the amount of imbalance (oz.-in.)
w = the weight (oz.)
Xf = the amplitude of vibration (mils)

Section 10.1 of the System Response chapter explains how Formula 6-5 is derived.

Let's assume we have a vibrating weight of 40,000 ounces (rotor weight plus 10% for the bearings and pedestals), an amplification factor of 3 (determined from the location of the operating frequency on the system response curve), and an initial peak-to-peak displacement of 10 mils. Since Formula 6-5 requires amplitude, we divide the peak-to-peak displacement in half and solve as follows:

$Ub = w * Xf$
$= 40{,}000 * .005$
$= 200$ ounce-inches

The trial weight can be estimated by the formula

$$Tw = \frac{Ub}{Af} \qquad \text{(Formula 6-6)}$$

where

Tw = the trial weight (oz.-in.)
Ub = the calculated imbalance (oz.)
Af = the amplification factor

Using our imbalance calculation of 200 ounce-inches in Formula 6-6, the estimated trial weight to be used in the trial weight run is

$Tw = Ub/Af$
$= 200/3$
$= 64$ ounce-inch trial weight

This is not only the estimated trial weight, but also the estimated actual imbalance in the rotor.

Since you know the approximate phase location of the imbalance, you would normally add the trial weight 180 degrees from this point. However, if a beat frequency is present and vector averaging is not used, add the trial weight at a location that will either keep the measured amplitude the same

or make it larger. This ensures that you will obtain an accurate phase measurement during the trial weight run.

You can bolt, clamp, glue, or tack weld a trial weight to the rotor. It is important that the weight stay on during the trial weight run; however, it will probably be necessary to remove it in order to make final correction. For purposes of illustration, we will add the trial weight at a location that will increase the amplitude of vibration. Assume that the rotor is a 6-bladed fan and that a 64 ounce-inch weight is added to blade number 4.

Once the trial weight has been added to the rotor, you can collect the trial weight run data. The operating frequency and the load conditions must be the same as those of the as is run. If it is not possible to maintain the same frequency, and if the operating frequency is close to *Fr*, then you might have to take data "on the fly" for both the as is and the trial weight runs. That is, take data while the operating frequency is changing, recording the vibration at the same frequency in both cases.

3.1.3 Calculate and Make Permanent Correction

The resultant vector *R* is the vector sum of the as is vector *A* and the trial weight vector *Tw*:

$$R = A + Tw \qquad \text{(Formula 6-7)}$$

where

R = the resultant vector
A = the as is vector
Tw = the trial weight vector

The vector *R* is measured as a displacement and a relative phase as before. Assume

$R = A + Tw$
= 18 @ 80 (that is, 18 mils at 80 degrees)

and solving for the trial weight,

$Tw = R - A$
= (18 @ 80) - (10 @ 30)
= 13.9 @ 114

In just one run the following information is available:

1. The system sensitivity is now known. Since 13.9 mils is equal to 64 ounce-inches, sensitivity= 64/13.9 = 4.60 ounce-inches per mil. This number is an indication of the system stiffness, and if it changes with time, the system has changed.
2. The trial weight phase of 114 degrees is blade number 4 on the rotor. If the system stiffness does not change, this phase relationship will not change and can be used in future balance operations on this same machine, along with the system sensitivity of 4.60 ounce-inches per mil.
3. Since the sensitivity is 4.60 ounce-inches per mil, the required correction weight is 10 mils (from the initial reading) times 4.60, or 46 ounce-inches.
4. Since the system sensitivity and the phase relationship are now known, it is possible to rebalance the rotor at some later date in just one run. To accomplish this, you must store the amount and the location of the trial weight, as well as the amplitude and the phase of the trial weight vector.

5. From the equation $Xf = Ub/W = 64/40{,}000 = 1.60$ mils amplitude, the free space displacement would be 3.2 mils. The actual displacement caused by the trial weight is 13.9 mils; thus the amplification factor *Af* is 13.9/3.2 or 4.34. This is a typical amplification factor, indicating that the balance procedure will not be difficult.

Looking at Formula 6-7,

$$R = A + Tw$$

it is apparent that if *Tw* were made equal to *-A*, then *R* would equal zero. Reducing the resultant vibration to zero is the ultimate aim of field balancing, and, in the single plane case is accomplished by adding a correction weight equal to *-A*. To set *Tw* equal to *-A*, you only need to divide the vector *-A* by the vector *Tw*, and multiply the result by the magnitude of the trial weight, thus

$$Correction = (-A/Tw) * Magnitude\ of\ Trial\ Weight \qquad \text{(Formula 6-8)}$$

In our example,

$$\begin{aligned} Correction &= [-(10 \text{ @ } 30)/(13.9 \text{ @ } 114)] * 64 \\ &= (.72 \text{ @ } 96) * 64 \\ &= 46.08 \text{ @ } 96 \end{aligned}$$

The required correction weight is 46 ounce-inches, which checks with the sensitivity calculations already done, and the location is 96 degrees from the position at which the trial weight was added to the rotor for the trial weight run. The calculation requires a +96 degree shift of the trial weight. However, the actual shift on the rotor depends on the method used to measure phase. Many a field balance exercise has failed simply because the weight was shifted in the wrong direction. Usually, positive calculated angles mean a shift opposite to the direction of rotation (called a **negative shift**), and negative calculated angles mean a shift in the direction of rotation (called a **positive shift**). The rules for weight shift are

1. If the reference mark used for phase was on the rotating member, and phase was read with a strobe light, then positive calculated angles mean negative angle shift on the rotor, and negative calculated angles mean positive angle shift on the rotor.
2. If the reference mark used for phase was on the stationary member, and phase was read from the rotor with a strobe light, then positive calculated angles mean positive angle shift on the rotor and negative calculated angles mean negative angle shift on the rotor.
3. If phase was read on a phase readout device driven by a reference transducer, then the technique used in the instrumentation to calculate the phase display will determine the direction in which to shift the weight.

Note that when using a strobe light the direction of rotation is not required to determine the direction in which to shift the weight. In this case, positive angles are in the direction of increasing angle, not the direction of rotation. If you are careful when taking data and performing calculations, then you do not need to add temporary weight in order to check the results. Not only are temporary weights unnecessary, they are unadvisable; they only add to the time and the stress on the machine.

You should not add correction weight at any location that will affect the rotor stiffness, such as the tip of an unsupported fan blade. Many times weight at the tip of such blades has reduced the blade resonance to the blade pass forcing function, and the fan fails in several months.

Add the correction weight so that it will not come off. If you use bolts and nuts, you should stake the threads (i.e. hit the threads with a center punch to prevent unscrewing) or use a thread locking compound on the threads.

If it is not possible to add the calculated correction at the correct location, you can split the weight between two possible locations so that the vector sum of the split weights is equal to the calculated correction vector. Several methods of splitting correction weight are discussed in Section 6.

3.1.4 The Final Run

Now that you have made a permanent correction, you can restart the machine for the final run. If the vibration level is now acceptable the machine may be left on. If the vibration level is not acceptable, consider this run to be a new as is run, and take data as before.

Do not remove the permanent correction weight, or make little correction changes in its mass or its location. Once "little biting" begins you have lost control. Simply use the new data as though it is a new problem, and repeat the balance procedure (using, of course, the information gained from the previous operation).

If the machine is running when the balance procedure is begun, then only two starts are necessary. Single plane balance should not take more than a maximum of four runs if the data is taken with care and accurate calculations are made.

4.0 PLOTTING SINGLE PLANE BALANCE CORRECTION

You can easily determine a single plane balance correction by using polar paper to plot the data. You should always plot vibration data on polar paper no matter how many correction planes there are. The plots serve as a road map during the procedure and give instant indication when the correction weight is moved in the wrong direction.

Figure 65 is a plot of the previous single plane problem. The as is vector is plotted as *a*, and the resultant vector, after the addition of the trial weight, is plotted as *r*. Connecting the ends of these vectors, from *a* to *r*, produces a vector *Tw* which is the trial weight vector. Its length is the magnitude of the vector, and its direction, from a to r, is the phase of the vector.

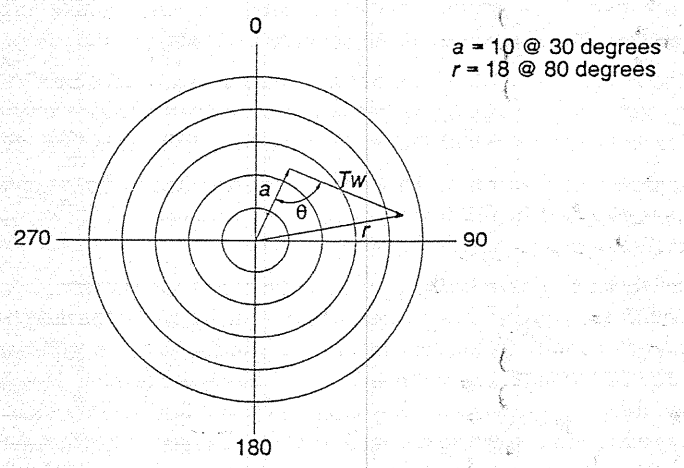


Figure 65 - *plotting a single plane balance solution*

The angle θ between the as is vector and the trial weight vector is the angle and the direction the trial weight must be moved to force the trial weight vector through zero. Dividing the length of the as is vector *a* by the length of the trial weight vector *Tw*, and multiplying the size of the trial weight by this number, gives the amount of correction weight to be added to the rotor.

The rules for the direction of weight shift on the rotor are the same as those discussed in the mathematical solution in Section 3.

Most calculator and computer balance routines include a program for single plane balance, and most multi-plane programs reduce to one plane. Since long-hand vector calculations are limited in accuracy, and since it is very easy to make a simple mistake, use a calculator or computer for all balance calculations. This becomes absolutely necessary when more than two correction planes are required for field balance.

5.0 MULTIPLE PLANE BALANCE

Since most rotor configurations do not lend themselves to single plane balance it becomes necessary to add correction weight in more than one correction plane. Although rules for determining whether the problem is a single plane or a multi-plane problem have been proposed for many years, such rules are a useless exercise. Data should always be taken at each bearing in the system, and the data will determine whether or not the problem is single or multi plane.

A typical rotor suspended between two bearings with an imbalance at any location along the axis of the rotor will cause vibration at both bearings. Since the rotor is not infinitely stiff, as the actual imbalance location moves from the bearing to the middle of the rotor, more of the imbalance force will be expended in bending the rotor than in moving the bearing.

In order to completely correct the rotor it would be necessary to add a correction weight directly opposite the imbalance location. Since this precise location would be very difficult to determine, multi-plane balance is accomplished by establishing two or more correction planes. Correction weights are calculated for each of these planes which result in an acceptable level of vibration.

For a rotor operating below the first critical there should be at least as many correction planes as there are bearings. Often the number of planes should equal the number of bearings plus one.

5.1 The Multiple Plane Balance Procedure

The multiple plane balance procedure is as follows:

1. Identify the correction planes
2. Catalog the correction planes in order of the mass moment of inertia at each plane
3. The as is run
4. The trial weight runs
5. Calculate and make permanent correction
6. The final run

5.1.1 Identify the Correction Planes

Multi-plane balance is a compromise, and the degree of compromise is dependent on the flexibility of the rotor. Since this procedure is a compromise, the location of the correction planes is very important. For a uniform rotor operating well below the first critical frequency, the correction plane location is mostly a matter of convenience.

If a uniform rotor is running above one or more critical frequencies, and N+2 planes cannot be used (where N is the number of shaft criticals), locate the correction planes 25% of the distance between the bearings, measured from the bearings. Use much greater care with non-uniform rotor configurations when you select the correction plane locations.

5.1.2 Catalog the Correction Planes

Label the correction planes in the declining order of the mass moment of inertia at each plane, rather than labelling them sequentially. Usually the plane with the highest mass moment of inertia is the most likely to be the area of difficulty. Many times it is possible to reduce the number of trial runs by first adding trial weights in the planes with the highest inertias.

The mass moment of inertia at the correction plane is proportional to the weight of the rotor concentrated at that plane times the square of the average radius of this mass. Even though the mass may be the same at two bearings, the mass at the greatest radius is the higher mass moment of inertia.

5.1.3 The As Is Run

This run is the same as that discussed for single plane, except that data is taken at each bearing. All of the precautions are the same as before, such as the repeatability of the data and the care in taking the data. This run results in an amplitude and a phase for each plane.

5.1.4 The Trial Weight Runs

A balance correction in one plane does not correct all the planes; in fact, balancing only one plane can often worsen the imbalance in other planes. A set of numbers, called **influence coefficients**, are used to determine how changes in one plane affect the other imbalance planes in the system. In order to calculate the influence coefficients for each plane, you must add trial weights in each of these planes. One run is required for each of these trial weight additions. You may remove the trial weight from one plane when adding a trial weight in the next plane, or you may leave the weight in the previous plane. They need not, and most likely should not, be the same for each plane, and need not be placed at the same angular position.

Calculate the amounts and locations of the trial weights used in each correction plane using the same procedure for the single plane procedure, and take the same care to insure that the weights will stay in place during the test runs.

The trial weight runs provide a resultant amplitude and phase for each of the correction planes so that you can calculate influence coefficients. The trial weight vector in the plane where you added the weight is the effect of the trial weight, and the resultant vectors in the other planes are called the **cross effect** vectors. With these coefficients you can solve the simultaneous equations for the imbalance correction required in each plane.

It is neither efficient nor advisable to add trial weights in each of the correction planes and take all of the data before making any correction calculations. In most cases, you can reduce the total number of runs required to achieve an acceptable running balance by making intermediate calculations.

Once you have selected the correction planes in the decreasing order of mass moment of inertia, add the first trial weight in plane A (the plane with the highest mass moment of inertia) and calculate a single plane correction for this plane.

In addition, calculate the resultant vibration from this correction weight in plane A for each of the other planes. You are finished if the vibration levels in each plane are within specification.

If the vibration at one or more planes is above specification, add the second trial weight in plane *B* and calculate a single plane correction for this plane along with the resultant in each of the other planes. Once again, if vibration is within spec the job is done. If the vibration is not acceptable, calculate a two plane correction for planes *A* and *B* and determine the resultants for the remaining planes.

Continue this process for planes *C*, *D*, etc. until the vibration in all planes is within the balance specification. For balance in four or more planes, you can usually obtain acceptable levels before using all of the correction planes.

5.2 Single Plane Correction for Plane A

If you use letters to identify the correction planes and numbers to identify the runs, then the as is run data consists of amounts and angles read from each of *n* bearings as *A1, B1, C1, D1, ...n1*. A trial weight is then added in plane *A* and the second run data is taken as *A2, B2, C2, D2, ...n2*. The single plane correction for plane *A* is calculated as before:

*Correction A = -A1/(A2-A1) * Magnitude of Trial Weight*

and the resultants in the other planes are

Resultant B = B1 + (B2-B1)(-A1)/(A2-A1)*

Resultant C = C1 + (C2-C1)(-A1)/(A2-A1)*

.

.

.

Resultant n = n1 + (n2-n1)(-A1)/(A2-A1)*

The plane *A* influence coefficients can also be calculated as

K11 = Plane A in Plane A = 1

K12 = Plane A in Plane B = (B2-B1)/(A2-A1)

K13 = Plane A in Plane C = (C2-C1)/(A2-A1)

.

.

.

K1n = Plane A in Plane n = (n2-n1)/(A2-A1)

5.3 Multiple Plane Solution

As discussed previously, add the trial weights in successive planes and perform intermediate calculations to determine the results in the other planes. Calculate the influence coefficients to determine the influence of the trial weight in the other planes. If you do not remove the trial weight from the previous plane, then you must use the trial weight vector in the calculations.

If you add trial weights in all correction planes, then you can determine all of the influence coefficients, so that the simultaneous equations for a zero-point correction (a correction to zero for one measurement point at each bearing) are:

*A1 = K11*Ua + K21*Ub + K31*Uc + ... + Kn1*Un*

*B1 = K12*Ua + K22*Ub + K32*Uc + ... + Kn2*Un*

$$C1 = K13*Ua + K23*Ub + K33*Uc + ... + Kn3*Un$$

.

.

.

$$n1 = K1n*Ua + K2n*Ub + K3n*Uc + ... + Knn*Un$$

where *Ua*, *Ub*, *Uc*, etc. are the imbalances in each of the correction planes, and can be determined by solving the matrix for each plane. For example, the imbalance in plane *A* is:

$$Ua = \frac{\begin{bmatrix} A1 & K21 & K31 & \dots & Kn1 \\ B1 & K22 & K32 & \dots & Kn2 \\ C1 & K23 & K33 & \dots & Kn3 \\ \dots & \dots & \dots & \dots & \dots \\ n1 & K2n & K3n & \dots & Knn \end{bmatrix}}{\begin{bmatrix} K11 & K21 & K31 & \dots & Kn1 \\ K12 & K22 & K32 & \dots & Kn2 \\ K13 & K23 & K33 & \dots & Kn3 \\ \dots & \dots & \dots & \dots & \dots \\ K1n & K2n & K3n & \dots & Knn \end{bmatrix}}$$

Use normal matrix manipulation to solve these equations. The denominator is always the same and need only be solved once. Calculate the imbalances in the other planes by moving the initial readings (*A1, B1, C1*, etc.) from column to column in the numerator.

If you are making a final correction by the addition of a correction weight, then add 180 degrees to the imbalance to indicate the light spot on the rotor. If you are correcting by removing weight from the rotor, then do not add 180 degrees to the answer.

The result of these calculations is a correction weight for each plane (expressed as a percentage of the trial weight added in the plane), and also an angle which is a shift from the trial weight position. Therefore it is important to record the amount of the trial weight used in each plane and the position at which this weight was added to the rotor. Many computer programs ask for the trial weight data so that the answers are in units of imbalance and the location is from some reference key on the rotor, rather than a percentage of the trial weight and a shift from the trial weight position.

If the rotor is flexible (running above one or more criticals), and if the number of correction planes can not be *N*+2, then the field balance procedure becomes more of a compromise. For example, if you chose two planes when you should have chosen five, you might have to add trial weights in phase at the uneven criticals, and out of phase at the even criticals. In this case you would take data at frequencies just below each of the criticals.

There are two possible solutions to the flexible rotor balance:

1. Assume that the rotor is linear at its operating frequency and calculate a zero-point correction for that frequency. This means, however, that the vibration levels at one or more criticals may be excessive during coastdown and startup. Also, a zero-point correction at one measurement point will not necessarily reduce the vibration at other measurement points on the same bearing.
2. Take vibration data just below each of the critical frequencies, adding trial weights as in the zero-point calculation. Then calculate a least squares compromise for the balance correction. Least squares calculations require a computer to perform the mathematics.

The difficulty with the least squares correction is that the balance condition at operating frequency is a compromise: it is not as good as it could be. The philosophy of the field balance expert and the vibration levels through criticals on coastdown and startup both determine whether a least squares fit is the best solution.

Many times balancing rotors which operate below their first critical also requires that the vibrations in more than one direction at each bearing (e.g. both the horizontal and vertical planes) be reduced to their minimum value. It is very difficult to balance a rotor so that the readings in all directions come down to zero. However, a compromise can be reached where each value is within acceptable limits.

In this case a least squares compromise between all measurement directions in each plane may be a better solution than a zero-point correction in one direction only (which has no regard for the effect on the vibration in the other directions). Only the test data can determine which of these techniques to use.

If you correctly identified the problem as imbalance, and took your data carefully, then you should perform the field balance in a minimum number of runs. As discussed before, permanently add the correction weights to the rotor, and if the balance is not acceptable, the final run becomes the as is run for the next procedure.

6.0 CORRECTION WEIGHT SPLIT

Many rotors have a limited number of possible correction locations, such as the blades of a fan or a series of drilled and tapped holes in the rotor. Since it is unlikely that the calculated balance correction weight location will coincide exactly with one of these possible correction locations, it is necessary to vector (or split) the correction weight between two or more possible locations. The vector sum of these split weights equals the correct balance weight at the correct location.

It is foolish to carefully determine the influence coefficients, the balance weight, and location required to balance the rotor only to throw a "little bit" here and a "little bit" there if it is not possible to add this weight at the calculated location. Equally dividing the weight between the two locations is also unacceptable. A correct weight split is not difficult, and it will help to eliminate unnecessary runs.

6.1 The Weight Split Procedure

Mathematically, the weight split procedure is as follows:

1. Choose two possible correction locations which lay on either side of the calculated correction location.
2. Let A equal the angle (in degrees) between these locations. This angle can either be calculated or measured with a protractor. If the possible correction locations are equally spaced around the rotor then A = 360 divided by the number of locations.

   Another way to calculate the angle A is to measure both the **chord** (the straight line distance between the two locations) and the radius of the correction locations. The angle A can then be calculated using the following formula:

$$A = 2 * asin\left(\frac{C}{2R}\right)$$

   where

A = angle between weight locations (degrees)
C = the length of the chord (inches)
R = the radius (inches)

asin is the arcsine function, which means "the angle whose sine is."

3. Let N equal the number of degrees from the nearest possible correction location to the actual calculated correction location.
4. Let W equal the desired balance correction.
5. The balance correction required in the nearest possible correction location is

$$Near\ Weight = W * \frac{\sin(A - N)}{\sin(180 - A)}$$

6. The balance correction required in the far (the other location) is

$$Far\ Weight = W * \frac{\sin(N)}{\sin(180-A)} \qquad \text{(Formula 6-9)}$$

Although the math is simple, there are many calculator or computer programs available which solve these equations. The field balance engineer need only enter the requested data in order to obtain a precise weight split. It is also possible to determine the weight split amounts by using polar paper. The procedure is very simple and, if carefully done, is quite accurate. Figure 66 is an illustration of this method.

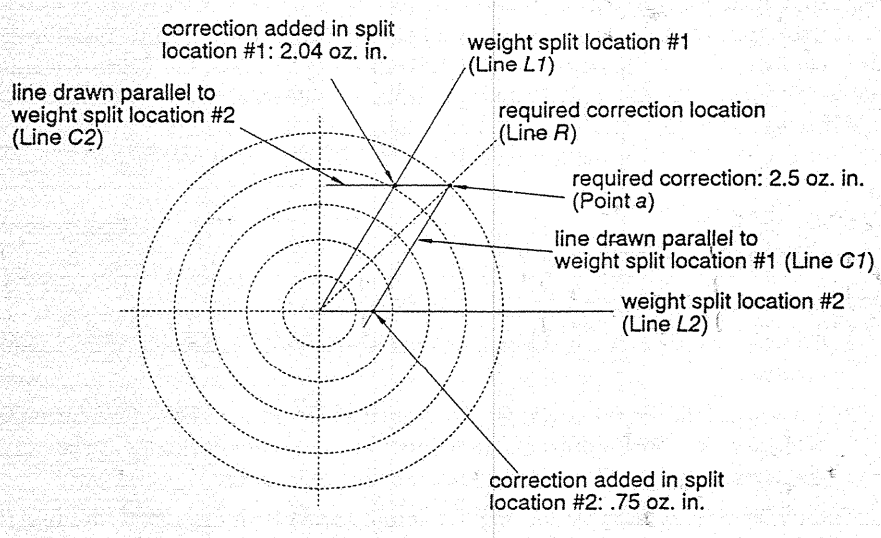


Figure 66 - *determining correction weight split using polar paper*

The polar paper procedure is as follows:

1. Draw a radial line on the polar paper at the angle of the calculated balance correction. Call this line R.

2. Scale the amount of the calculated balance correction on this radial line. Call this point *a*.
3. Draw a radial line on the polar paper at the angular location of one of the possible weight split locations. Call this line *L1*. Draw another radial line at the location of the second possible weight split location, and call this line *L2*. Lines *L1* and *L2* should span the required correction location.
4. Place your pencil on point *a*. Draw a line parallel to line *L1* until it intersects line *L2*. Call this new line *C1*.
5. The intersection of line *C1* with line *L2* is the scaled amount of the weight to be added at location *L2*.
6. Repeat Steps 4 and 5 for the other location, drawing a line *C2* from point *a* until it intersects line *L1*. This intersection represents the scaled amount of the weight to be added at location L1.

As shown in Figure 66, it is possible to calculate a weight split by using polar paper and drawing only five lines.

7.0 FOUR RUN BALANCE WITHOUT PHASE

In the rare case where you cannot obtain a phase reading from the rotor during a field balance operation, you can balance the rotor by reading only the amplitude of the test runs.

This procedure requires two extra runs for each plane and considerably more mathematics, plus extreme care in taking the data, so it is not practical for much more than single plane balance.

7.1 Single Plane Balance Without Phase

The steps for a single plane balance operation without phase are as follows:

1. Read the initial amplitude. This amplitude must be the first order (the amplitude at the rotating frequency of the rotor). It will probably be necessary to use filtered readings in order to obtain acceptable results. If you use filtered readings, take extreme care to compensate for any phase shift in the filter.
2. Mark off three locations on the rotor in which to add a trial weight. The mathematics are less complicated if these are equally spaced locations and if the trial weight is kept the same for each of the trial weight runs.
3. Read the first order amplitude with the trial weight in location 1, then in location 2, and finally in location 3.
4. Calculate the amount and location of the balance correction weight either mathematically or by the use of polar paper.

The equations for the simple case, where the same trial weight is used at three equally spaced locations and at the same radius, are

$$2 * A0 * TW * \cos(UA) + TW^2 = A1^2 - A0^2 \qquad \text{(Formula 6-10)}$$

$$1.7*A0*TW*\sin(UA)-A0*TW*\cos(UA)+TW^2=A2^2-A0^2 \qquad \text{(Formula 6-11)}$$

$$1.7*A0*TW*\sin(UA)+A0*TW*\cos(UA)-TW^2=A0^2-A3^2 \qquad \text{(Formula 6-12)}$$

where

UA = The angle of the original imbalance
TW = The trial weight in amplitude units
A0 = The initial amplitude
A1 = Amplitude with the trial weight at location 1
A2 = Amplitude with the trial weight at location 2
A3 = Amplitude with the trial weight at location 3

Solving Formula 6-9 for cos(*UA*) gives

$$\cos(UA)=\frac{A1^2-A0^2-TW^2}{2*A0*TW} \qquad \text{(Formula 6-13)}$$

Substituting Formula 6-12 in Formula 6-10 and solving for sin(*UA*) gives

$$\sin(UA)=\frac{X}{1.7*A0*TW} \qquad \text{(Formula 6-14)}$$

where

$$X=A2^2+.5*A1^2-1.5*A0^2-1.5*TW^2$$

Substituting Formula 6-13 in Formula 6-11 and solving for *TW* gives

$$TW=\sqrt{\frac{A1^2}{3}+\frac{A2^2}{3}+\frac{A3^2}{3}-A0^2} \qquad \text{(Formula 6-15)}$$

and

$$UA=atan\left(\frac{2*X*A0*TW}{1.7*A0*TW*(A1^2-A0^2-TW^2)}\right) \qquad \text{(Formula 6-16)}$$

The balance correction amount is then

$$Correction\ Amount = (A0/TW) * Amount\ of\ Trial\ Weight \qquad \text{(Formula 6-17)}$$

and the correction angle is

$$Correction\ Angle = UA + 180 \qquad \text{(Formula 6-18)}$$

Computer and calculator programs are available to solve these equations even if the trial weight locations are not equally spaced. Since there are three unknowns, it is necessary to add the trial weight at at least three locations in order to arrive at a discrete solution.

The atan (arctangent) is used so that the calculated angle is in the correct quadrant. Every possible tangent value corresponds to two angular values, 180 degrees opposite each other. For example, .0875 is the tangent of both 5° and 185°. So when you use the atan function, make sure that your answer lies in the correct quadrant.

Polar paper provides an easy method to solve these equations if a computer is not available. This procedure, illustrated in Figure 67, is as follows:

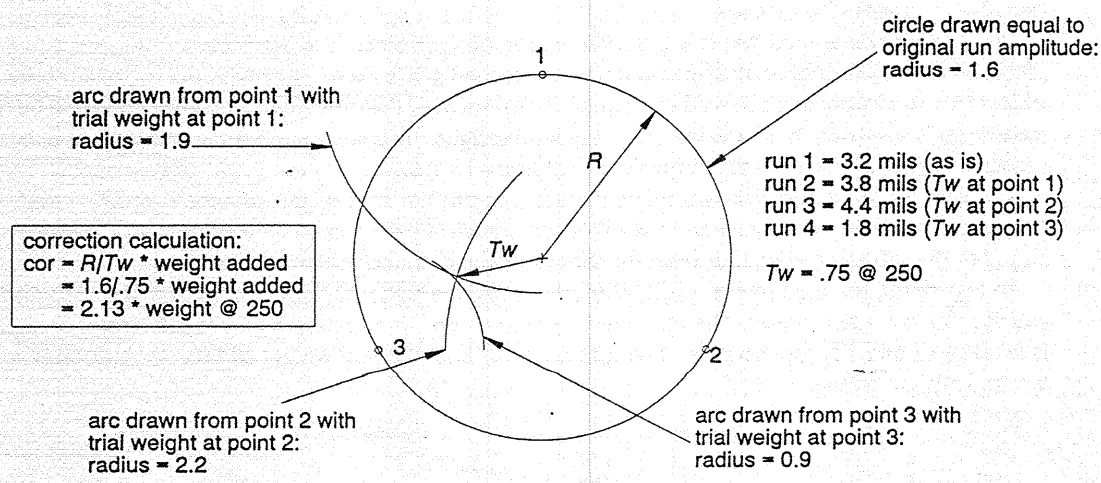


Figure 67 - *solving four-run formulas on polar paper*

1. Take the amplitude readings with and without the trial weight as described in the mathematical solution.
2. Draw a circle on the polar paper with the center at the center of the polar plot and the radius scaled to the measured amplitude without a trial weight. In the illustration, the radius was set equal to ½ the measured amplitude.
3. Mark points on this circle which correspond to the locations on the rotor where the trial weight is added. Zero degrees on the circle will be zero degrees on the rotor. In the illustration, points 1, 2, and 3 are equally spaced around the circle.
4. Using the same scale as Step 2, draw an arc whose radius is equal to the amplitude measured with the trial weight at location 1, and whose center is located at point 1 on the circle on the plot.
5. Repeat Step 4 for points 2 and 3, using the amplitude measured with the trial weight located at each of these positions.

If the measured amplitudes are filtered at first order, and the data is taken carefully, these three arcs should intersect in a point. If there is noise coupled with these readings, the arcs will inscribe an area. The vector from the center of the polar plot to the center of this intersection (or area) represents the magnitude of the trial weight in amplitude units, and the angle is the angle at which the correction weight is to be added.

The correction amount is the radius of the circle on the polar plot, divided by the length of the trial weight vector, then multiplied by the weight of the trial weight. This weight is to be added to the rotor at the angle of the trial weight vector.

8.0 PHASE READINGS USING A REFERENCE TRANSDUCER

It is always more accurate, and usually much easier, to read phase angles during a field balance exercise using a reference transducer.

This transducer may be a magnetic pickup looking at a key or some other magnetic material which may be used as a reference point on the shaft or the rotor, or a photo pickup and a piece of light reflecting tape. There is no case where a strobe light can be used to determine phase that a reference transducer cannot also be used.

It is sometimes argued that it is possible to save one run since it is not necessary to shut down to prepare the rotor for phase measurement when a strobe light is used. However, the use of a reference transducer allows phase to be read so accurately that most of the time more runs are eliminated by the transducer's accuracy than are saved by the strobe light. It is also possible in many field balance problems to set up the reference transducer without a shutdown.

Phase is the time between two related events, and in the case of the balance exercise these two events are the reference signal and the vibration signal. Phase can be measured from the reference signal to the vibration signal, or from the vibration signal to the reference signal.

In Figure 68, the time (phase angle) from the reference signal to the vibration signal is labeled *R1* and *R2* (for run 1 and run 2). The time (phase angle) from the vibration signal to the reference signal is labeled *V1* and *V2*, for two runs. The first run is without the trial weight and the second run is with the trial weight added.

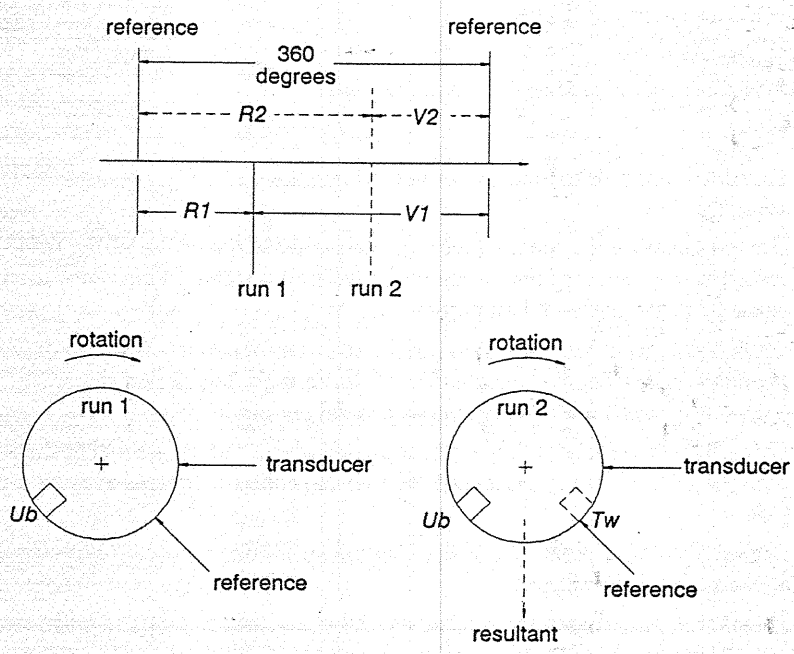


Figure 68 - *phase between reference and vibration signals*

Figure 68 shows that the first case results in a larger angle; a positive increase in phase occurs when the trial weight is added. Yet the trial weight must be moved in a negative phase direction to balance the rotor.

In the second case, the addition of the trial weight results in a smaller angle; a minus or decrease in phase occurs after adding the trial weight. The trial weight must also be moved in the minus direction.

It is very important to know which technique is employed in the instrumentation used for the balance procedure. In most cases phase is measured from the reference signal to the vibration signal so that a positive calculated shift means a negative shift on the rotor, and a negative calculated shift means a positive shift on the rotor.

If the instrumentation measures phase from the vibration signal to the reference signal, then positive calculated angles mean a positive shift, and negative calculated angles mean a negative shift. See the chapter on Phase for a complete discussion on weight shift.

SUMMARY

Balancing rotating assemblies in the field is not only efficient, but it is many times a necessary procedure in order to obtain an acceptable running balance.

Field balancing should never be attempted until a complete vibration analysis of the machine has determined that the forcing function is rotor imbalance. In addition, the rotor resonance, bearing support resonance, and other system resonances should be investigated to insure that the operating frequency is at least 20 (better 30) percent above or below these resonances.

Rotors can be balanced in a single plane or in multiple planes. Accurately recording the amount and angle of imbalance in each plane, as well as the amount and location of all trial weights, is essential.

If it is not possible to add a correction weight at the calculated location, the weight can be split between two locations. The vector sum of these split weights equals the correct balance weight at the correct location.

If phase cannot be obtained from the rotor during field balancing, the rotor can be balanced by reading only the amplitude of the test runs. This procedure requires two extra runs for each plane and considerably more mathematics, and is not practical for more than single plane balance.

Phase can be read from either a strobe light or a reference transducer. Using a reference transducer is always more accurate and often times much easier than using a strobe light. Instrumentation may read phase from the reference signal to the vibration signal or vice versa. It is important to know which technique is used by your instrumentation, as this affects your determination of correction weight placement.

7 SHAFT ALIGNMENT

Working at an industrial facility on a large coupled machine, the authors, in analyzing the data, determined that shaft center lines were not properly aligned. When this information was made known to the plant maintenance manager, he said he would have his alignment expert check the machine. The expert arrived with a carpenter's square and a yardstick. When the authors asked him how he planned to proceed, he said that he was going to check the alignment on that machine with the square and the yardstick. The expert stated that he had been aligning machines this way for 25 years.

Industry in general is just beginning to understand the importance of alignment and how it should be properly performed. However, there are still many cases where those responsible for machine installation and maintenance do not understand both the importance of shaft alignment and the methods of properly aligning shafts.

1.0 INTRODUCTION

The alignment of shaft center lines on coupled machines (sometimes called **coupling alignment**) is one of the most important aspects of proper machine installation. A popular fallacy that has existed for many years is that flexible couplings will compensate for any reasonable amount of shaft center line misalignment. Although there are generally accepted standards for shaft misalignment with various types of flexible couplings, it is best to have the shaft center lines aligned as closely as possible. Table 1 lists suggested alignment tolerances as they relate to shaft RPM.

There are some cases where a little misalignment is required. For example, some slight misalignment is necessary in order to properly lubricate the gear teeth in a gear coupling. However, it is important to have the shafts of coupled machines aligned very closely under operating conditions of temperature and load.

It is not the purpose of this chapter to teach the principles of shaft alignment, although some alignment techniques and precautions will be reviewed. Instead, this chapter focuses on the vibrations

Alignment Condition	Rotational speed	Excellent tolerance	Acceptable tolerance
Soft foot		2.0 mils	3.0 mils
Parallel misalignment	<1000 RPM	4.0 mils	5.0 mils
	1200 RPM	3.0 mils	4.0 mils
	1800 RPM	2.0 mils	3.0 mils
	3600 RPM	1.0 mils	2.0 mils
	>5000 RPM	0.5 mils	1.0 mils
Angular misalignment	<1000 RPM	1.0 mils/in.	1.5 mils/in.
	1200 RPM	0.5 mils/in.	1.0 mils/in.
	1800 RPM	0.3 mils/in.	0.5 mils/in.
	3600 RPM	0.25 mils/in.	0.3 mils/in.
	>5000 RPM	0.2 mils/in.	0.25 mils/in.
Coupling with spool piece (parallel misalignment per inch of spool piece length)	<1000 RPM	1.5 mils/in.	2.0 mils/in.
	1200 RPM	1.0 mils/in.	1.5 mils/in.
	1800 RPM	0.5 mils/in.	1.0 mils/in.
	3600 RPM	0.3 mils/in.	0.5 mils/in.
	>5000 RPM	0.25 mils/in.	0.3 mils/in.

Table 1 - *suggested alignment tolerances*

that result from shaft center line misalignment in coupled machines. If misalignment is not the most common vibration forcing function in industry today, it is in second or third place.

2.0 TYPES OF MISALIGNMENT

Shaft center line misalignment may be one of two types: **parallel** (sometimes called **offset**) or **angular**. Figure 69 shows an example of both types of misalignment.

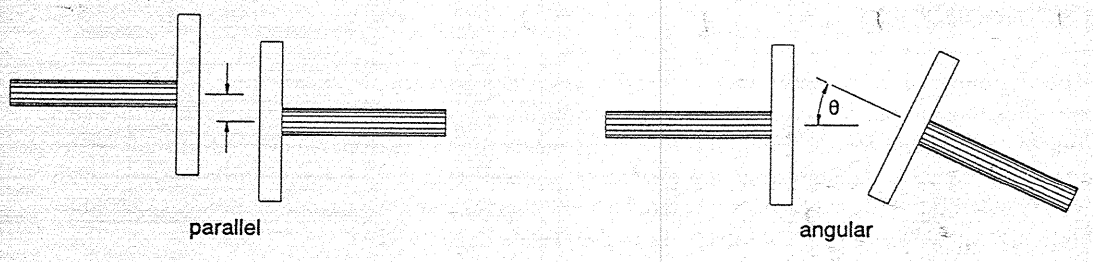


Figure 69 - *types of misalignment*

Angular misalignment occurs when the shaft center lines meet at an angle. Parallel misalignment occurs when the shaft center lines are parallel, but are offset from each other. The center lines may be offset horizontally, vertically, or a combination of the two. In fact, almost all shaft center line misalignment problems are a combination of both parallel and angular misalignment.

Another type of misalignment not associated with couplings is **bearing misalignment**. The center lines of two coupled shafts may be properly aligned, but the bearings on one side of the coupling may be misaligned. Bearings may be misaligned if they are not mounted in the same plane, if they are not normal to the shaft (i.e. they are cocked relative to the shaft), or if the machine distorts due to a soft foot, an uneven base, or thermal growth.

3.0 ALIGNMENT PROCEDURES

The **Reverse Dial** and **Face and Rim** methods (two of the most popular methods of alignment) both use dial indicators or laser sensors to measure the misalignment between the shafts. Brackets are mounted on each side of the coupling, and each bracket holds an indicator rod that spans the coupling. The dial indicators, as shown in Figure 70, can be mounted on the rod or on the bracket with the rod resting on the indicator button.

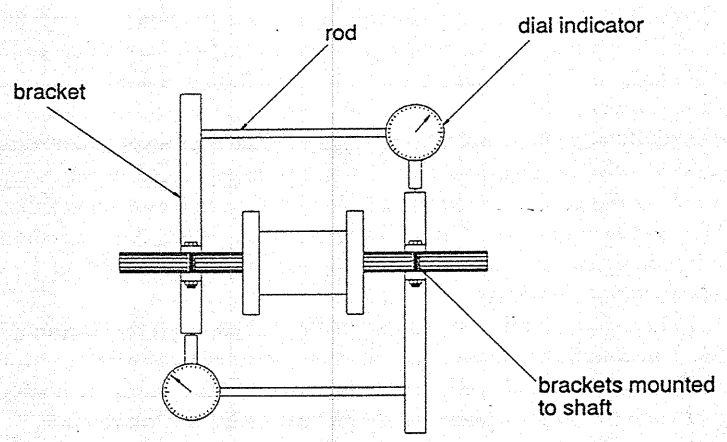


Figure 70 - *using dial indicators for alignment*

Many systems mount the dial indicator on the rod, which greatly increases the rod sag. Section 4.6 of this chapter discusses dial indicator sag in more detail. It is often more practical to mount the dial indicator on the bracket so that its weight does not add to the rod sag. The brackets should be as far apart as possible in order to get the most accurate dial indicator readings. Mounting the brackets as high above the shaft center line as possible improves accuracy for the face and rim method.

If you imagine the shaft or coupling face as a clock, the dial indicators are usually zeroed at the 12:00 position and readings are taken at the 12:00, 3:00, 6:00 and 9:00 positions as shown in Figure 71(a). However, it is not always possible to take readings at these positions due to clearance problems. For example, the coupling may be too close to the floor, or a pipe or support member underneath the shaft may prevent you from rotating the shaft through the 6:00 position.

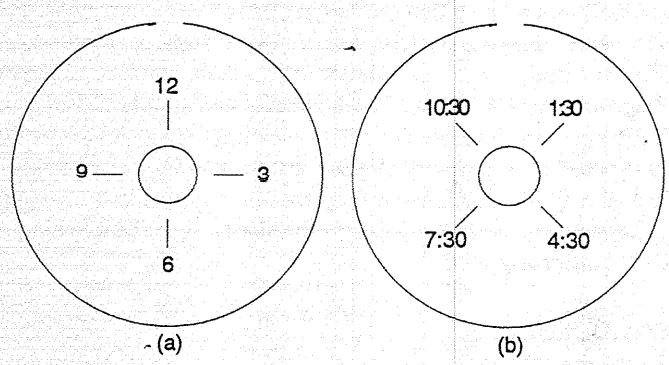


Figure 71 - *positions for taking dial indicator readings*

In this case you may still be able to take four equally-spaced readings, but at other positions: the 1:30, 4:30, 7:30, and 10:30 positions, for example, as shown in Figure 71(b). These readings may be used directly in the alignment calculations and usually result in alignment within specifications.

Or you may take readings at the 12:00, 3:00, and 9:00 positions and use the "validity rule" to calculate the 6:00 reading. The "validity rule" states that the sum of the left and right readings is equal to the sum of the top and bottom readings. Thus if readings are taken at the 12:00, 3:00, and 9:00 positions, the 6:00 reading is equal to the sum of the 3:00 and 9:00 minus the 12:00 readings.

When it is not possible to rotate the shaft through a full 360 degrees, either of these methods of taking data usually allows the operator to align the shafts to acceptable tolerances.

As a general rule the coupling should **not** be disassembled when using dial indicators to read misalignment. If the shafts are uncoupled they are free to move axially during rotation (called **end float**), which can cause invalid dial indicator readings. Also, the shaft should *always* be rotated in the same direction each time data is taken. Otherwise, any backlash in the coupling will introduce dramatic errors in your readings.

Some coupling designs allow minimal end float even when the coupling is assembled (both gear and metallic grid couplings, for example, are not restricted from axial movement). This end float can be ignored if the reverse dial method is being used. If face and rim readings are being taken it is important to force the shafts against the thrust bearing during the rotation.

Also, breaking the coupling makes it difficult to rotate both shafts simultaneously, which is essential for accurate misalignment readings. When the coupling is fully assembled, both shafts are rotated together so that the dial indicator stem remains fixed at one point throughout the rotation of the shaft.

However, if the coupling has been disassembled and only one shaft is rotated, then the dial indicator stem will travel completely around the coupling hub during shaft rotation. If the coupling is even slightly out of round (no coupling is perfectly round), the eccentricity of the coupling circumference will add to the dial indicator readings as misalignment.

There are some cases when you must uncouple the machines. For example, suppose you are aligning a 200 hp motor coupled to a gear box that drives a mining conveyor belt ½ mile long. Given the mass (and weight) of the driven machine it would be impossible to manually rotate both shafts during alignment. In cases like these you have no choice but to disassemble the coupling and rotate only one shaft. The alignment will not be ideal, since you now have no way to compensate for coupling runout, but in cases like these you have little choice.

3.1 The Face and Rim Method

The face and rim method is the oldest of the two dial indicator methods discussed here. A bracket holding two dial indicators is fastened to the shaft on one side of the coupling, and the dial indicators are positioned against a bracket mounted to the shaft on the other side of the coupling. The coupled shafts are turned and the dial indicator readings are recorded at the four positions. Figure 72 illustrates two shafts being aligned with the face and rim method.

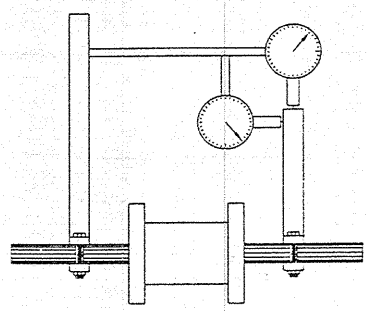


Figure 72 - *the face and rim method*

The face and rim method can be used on either side of the coupling. The indicator bracket may be mounted on the fixed machine and read off the movable machine or vice versa.

The brackets used in the face and rim method should be as far apart and as high above the shaft center line as possible in order to obtain accurate face readings. If the dial indicator used to take face readings was mounted close to the shaft center line, any movement of the shaft center line would only deflect the coupling hub very slightly. Consequently, it is more difficult to obtain dial indicator readings which adequately represent the movement due to misalignment.

This is illustrated in Figure 73, which shows an angle θ containing two line segments *a* and *b* (line segments *a* and *b* are said to be **subtended** by the angle). Since line *a* is close to the vertex of angle θ, the length of *a* is rather short. Line *b*, however, is longer since it is farther from the vertex. In fact,

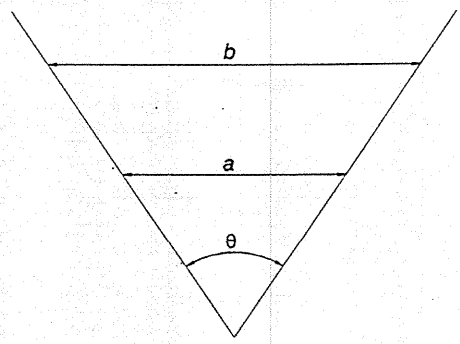


Figure 73 - *length of subtended line segments*

line segments like *a* and *b* that join the sides of angle θ will get longer and longer the farther away they are from the vertex. This is why it is rather difficult to aim a telescope at the moon: even a slight movement of the telescope subtends a very long line 248,000 miles away! By using long brackets, you position the dial indicator farther away from the shaft center line. Movement of the coupling face subtends a longer line, and the dial indicator readings are more accurate.

The problem with long brackets, however, is that they have a tendency to deflect due to the side load of the rod and indicator, which adds to the sag. The rod that holds the dial indicator also sags due to its own weight and the weight of the dial indicator.

Rod sag can be reduced by mounting the dial indicator on the bracket instead of the rod, as shown in Figure 74. The dial indicator is positioned so that the stem points away from the shaft and touches the rod. The bracket deflection can also be reduced by using a counter-weight to offset the side load due to the rod and indicator.

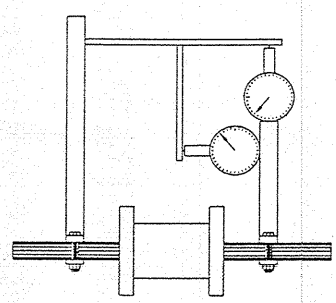


Figure 74 - *mounting the dial indicator on the bracket*

Bracket and rod sag are sources of error in dial indicator readings, and are explained in Section 4.6 of this chapter.

3.2 The Reverse Dial Method

The reverse dial method is a simpler and more accurate method of measuring shaft center line misalignment, and is becoming much more popular than the face and rim method. Brackets are mounted on each shaft, on both sides of the coupling. One bracket holds a rod which spans the coupling and rests on a dial indicator on the opposite bracket. Another rod and indicator reverse-span the coupling as shown in Figure 75.

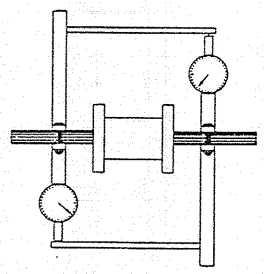


Figure 75 - *the reverse dial method*

Readings from the reverse dial method will be more accurate if the length of the rod between the brackets is as long as possible. If the coupling hubs are joined by a long spool piece, it may not be possible to fashion a bracket and rod that spans the entire length of the spool piece. In this case, position the dial indicator stem against the surface of the spool piece as shown in Figure 76.

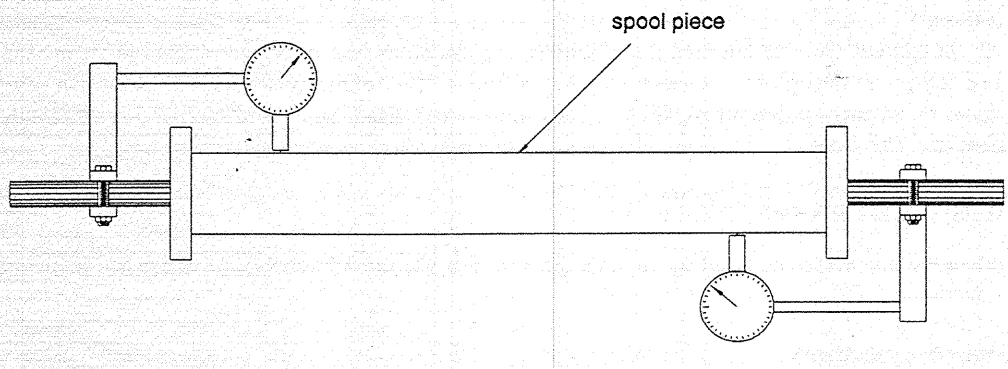


Figure 76 - *taking reverse dial readings off of a spool piece*

3.3 Other Methods

Several newer methods have been developed to read misalignment. One technique that is becoming very popular uses a laser beam instead of an indicator and bracket. A laser emitter and detector is mounted on one side of the coupling, and a reflector is mounted on the other side.

Before rotating the shafts the reflector is adjusted so that the laser beam is centered in the detector. As the shafts are rotated and misalignment bends the coupling, the beam will be deflected off of the center of the detector. A small computer attached to the detector measures the beam deflection.

Lasers are highly accurate measuring devices, can usually be mounted quickly, and can accommodate very long coupling spans without worrying about sag. However, they are costly systems, requiring periodic recalibration at the factory. Also, high heat and/or steam can affect the optical properties of the laser beam/detector combination.

One of the newer systems uses two three-dimensional position sensors that are mounted vertically to the shaft on both sides of the coupling. After mounting, each sensor lies along the center line of its shaft. The two sensors are then joined by a solid rod, and the dimensions of both machines are input to a computer. The sensors send their position information to the computer, and the misalignment corrections can be displayed both numerically and pictorially.

The biggest advantage of such a system is that the shafts do not have to be rotated; the position sensors detect misalignment off of the rod that connects the two. These systems are easy to install, provide real-time feedback during the alignment process, and are fast: setup and readings can often be obtained in three to five minutes. Such a system would be ideal for aligning the motor-driven gear box connected to the mining conveyor mentioned in Section 3. If adequate clearance is available, the shafts could be aligned without breaking the coupling or rotating both shafts.

It is important when using a position sensor system to be able to obtain the shaft diameters and distances to the inboard and outboard feet precisely. Very small errors in the shaft diameters can cause large errors in the alignment calculations. It is also necessary to have a standard in order to set up and calibrate the instrument. One more word of caution: these systems are very sensitive to temperature changes during the calibration and alignment process.

Unlike dial indicators or laser techniques, this instrument cannot detect a bowed shaft unless readings are taken at positions which are 90 degrees apart. Also, since the sensors must be mounted directly to the shaft, it cannot be used unless there is enough shaft exposed between the coupling and the machine.

With the advent of laser alignment techniques, sag becomes less important since the laser beam does not sag (even though the brackets may sag a little). The tendency seems to be to move more toward laser alignment procedures. There are advantages and disadvantages to each of these methods, and these must be carefully weighed when you choose a method for each alignment job.

4.0 ALIGNMENT PREPARATION

There are many steps in preparing for an alignment job, the most important of which are reviewed in this section.

4.1 Base Preparation

Machine bases must be properly cured, thoroughly cleaned, and prepared for installation of the machines. If the bases are old, the area around the feet must be thoroughly cleaned on both the movable and the fixed machine. In some cases, it may be necessary to re-machine or modify the base. Bases should be checked for cracks, bows, or any other area of weakness which may hinder proper alignment or allow the alignment to change.

4.2 Soft Feet and Frame Twists

Use dial indicators to check for soft feet, and ensure that both the fixed and movable machines are properly shimmed to prevent a soft foot or a twist in the machine frame. Mount a dial indicator on the sole plate with the stem against the machine foot as shown in Figure 77. Loosen the hold-down and check for any rise in the machine foot.

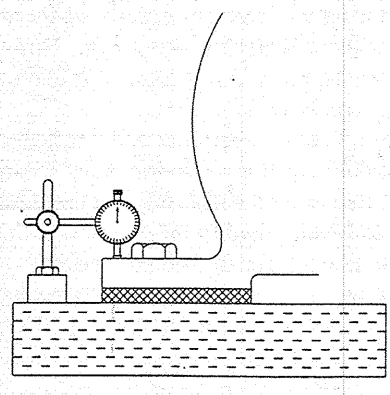


Figure 77 - *using dial indicators to check for a soft foot*

If the foot rises more than 1 mil, insert a shim whose thickness is equal to the rise. Retighten the bolt and recheck the foot. When the foot is shimmed so that there is no change greater than 1 mil, tighten the bolt to specification. Repeat this procedure for each machine foot in the tightening sequence established in Section 4.3.

The base of the machine foot must be parallel with the sole plate. If it is not, the soft foot check described previously becomes an exercise in frustration: shimming one foot causes another foot to distort. Shimming this foot distorts a third, and so on forever around the machine. Even worse, the frame of the machine is twisted each time the hold-down is torqued to specification. Both of these conditions not only seriously affect the alignment process, but may be the cause of premature failure.

At worst, the machine base or the sole plate may need to be machined so that feet and sole plate are flat and parallel to each other. At best it is necessary to use tapered shims at the non-parallel foot.

Shims should be made of clean, flat material, free of burrs, and they should be made of a material such as stainless steel to prevent rust. As few shims as possible should be used under each foot; it is not wise to use many thin shims which could result in a springy or spongy shim pack.

4.3 Tightening Machine Foot Bolts

There should be a tightening sequence on the bolts of each machine foot, and that sequence should be followed throughout the alignment procedure: bolts are tightened according to the sequence and are loosened in the opposite order.

4.4 Lifting and Moving the Machine

Do not lift the machine any more than necessary when inserting shims for vertical movements. Use jack screws for both vertical and horizontal movements. If jack screws are not available, then weld brackets to the machine base so that you can move the machine horizontally. **Never move a machine with a sledge or hammer!**

4.5 Piping Connections

Align the machine with all piping and other connections disconnected. Then use dial indicators to ensure that no machine movement occurs when the piping or other services are reattached.

The machine being aligned should **never** be the support for these services.

4.6 Dial Indicator Sag

When dial indicator brackets and rods are used, their sag should be carefully measured and entered into the misalignment readings. Neglecting sag is one of the most common errors encountered in shaft alignment. The sag of the indicator bracket and rod combination can be easily measured by mounting the brackets you will use during alignment to a piece of pipe as shown in Figure 78. Both brackets must be mounted exactly as they will be mounted to the shafts: the same height, same rods, same button-to-button distance, etc.

The dial indicator is set to zero at the top and the entire assembly is turned through 180 degrees to measure the bracket/rod sag. When the dial indicator is zeroed at the top, gravity is pulling the bracket and the rod down toward the pipe. However, when the pipe is rotated 180 degrees, gravity is now pulling the dial indicator away from the pipe. Consequently, sag readings will be negative numbers, but they should be used as positive numbers in the misalignment equations (i.e. a reading of -4 would

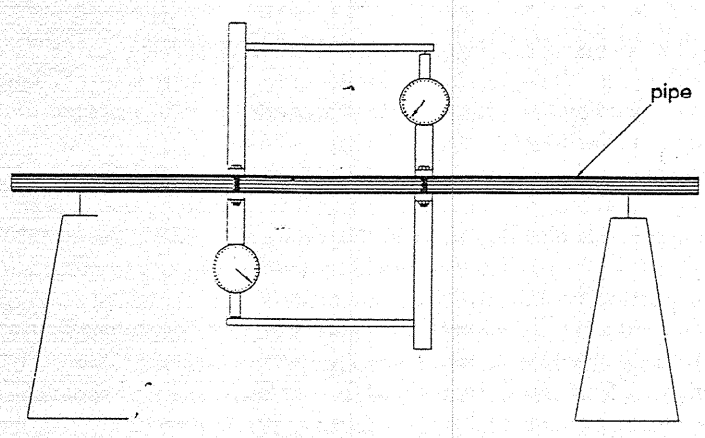


Figure 78 - *measuring indicator bracket sag*

be entered as 4). Since we are subtracting a negative number from the dial indicator readings, sag becomes a positive number in the equations.

If the face and rim method is used, the rim sag will be a negative number, but the face reading may be either negative or positive. Most people ignore the face sag and use the rim sag as in the reverse dial method. This is acceptable since most machine alignment requires more than one move, and the face sag becomes less important as the alignment improves. If a pipe and bearing stands are not available, the bracket sag can be estimated. This method gives fairly usable readings but is not as accurate as measuring the sag. For round indicator rods, the sag Sg is found by the formula

$$S_g = 2.829 * 10^{-5} * L^3 * \left(\frac{K_r}{D^4 - d^4} \right) \qquad \text{(Formula 7-1)}$$

where

Sg = Dial indicator sag (mils)
L = Length of rod (inches)
D = Outside diameter (inches)
d = Inside diameter (inches)
Kr = Spring constant of round rod

Formula 7-2 is used to find the spring constant Kr:

$$K_r = 1.334 * \left(D^2 - d^2 \right) * L + W_d \qquad \text{(Formula 7-2)}$$

where

Kr = spring constant of rod
Wd = Weight of dial indicator (ounces)
L = Length of rod (inches)
D = Outside diameter (inches)
d = Inside diameter (inches)

For square indicator rods, the sag *Sg* is found by the formula

$$S_g = .833 * 10^{-5} * L^3 * \left(\frac{K_s}{D^4 - d^4} \right) \qquad \text{(Formula 7-3)}$$

where

Sg = Dial indicator sag (mils)
L = Length of rod (inches)
D = Outside diameter (inches)
d = Inside diameter (inches)
Ks = Spring constant of square rod

Formula 7-4 is used to find the spring constant *Ks*:

$$K_s = 1.699 * \left(D^2 - d^2 \right) * L + W_d \qquad \text{(Formula 7-4)}$$

where

Ks = spring constant of rod
Wd = Weight of dial indicator (ounces)
L = Length of rod (inches)
D = Outside diameter (inches)
d = Inside diameter (inches)

4.7 Thermal Growth

Rotating and reciprocating equipment heats up during operation. The major sources of heat are from the process (for example, heat generated as gases are compressed) and from friction (bearings, piston rings, etc.).

All metals expand as their temperature increases. This change in length, called **thermal growth**, can cause the shaft center lines to shift from their cold positions. In some cases this shift is minimal, and the cold alignment is satisfactory for operation at normal load and temperature.

In other cases, however, thermal growth can be extensive enough to cause serious misalignment problems. In the case of pumps and compressors, the attached piping undergoes thermal growth as well, which may put increased pipe strain on the system.

The change in temperature is rarely uniform throughout a machine. For example, the areas of combustion or injection in turbines and exhaust manifolds on engines and reciprocating compressors are much hotter than other areas on the machine. This uneven heating causes the machine to pitch and bend rather than grow straight up.

Note that we are primarily concerned with vertical growth. When a shaft is centered in a machine, the metal on the left of the shaft grows to the left, the metal on the right grows to the right, and the shaft remains centered. There is very little thermal movement side to side unless the shaft is not centered (for example, in a gearbox) or there is uneven heating as discussed before. The metal below the shaft tries to expand downward, but it cannot since the machine is sitting on a solid foundation. Therefore the thermal growth pushes the shaft upwards.

It is very important to know the thermal growth characteristics of the system you are aligning. Thermal growth can be estimated at 6 micro-inches (10^{-6} inches) per inch of steel per degree

Fahrenheit rise from ambient temperature. But you must be aware that this method of estimating thermal growth is inaccurate; **it is always best to measure the thermal growth instead of estimating it!** It is also unwise to use the manufacturer's figures instead of measuring thermal growth, since the manufacturer's figures are most likely estimates as well. Thermal growth can be measured very accurately by laser equipment, inside micrometers and tooling balls, or by special alignment bars or water columns.

Using inside micrometers and tooling balls is the least expensive method of measuring thermal growth. These are telescoping bars of stainless steel with a large dial indicator readout in the middle. As the smaller shaft slides in and out, its position is recorded on the dial. The bars are calibrated before each alignment job by using a standard made from a metal called Invar. Invar has very little thermal growth; it expands only .001 inch per 100 degrees Fahrenheit.

Measurement is very simple. Studs with a ball top (called **benchmarks**) are permanently mounted to the machine foundation and bearing housing. Once the shafts are aligned, the bars are placed between the benchmarks as shown in Figure 79 and the cold readings (i.e. before startup, with the machine off and cold) *A* and *B* (and angles θ and ϕ) are taken.

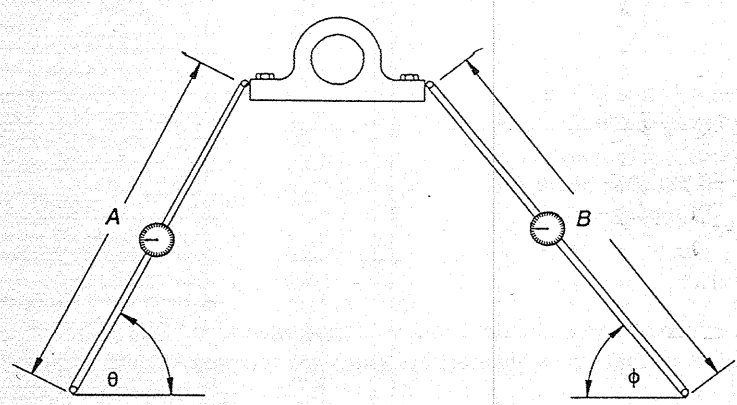


Figure 79 - *measuring thermal growth*

Bring the machine online once the cold readings are taken. Measure the lengths A and B again once the machine reaches normal operating load and temperature. You can then use a simple mathematical (or graphical) technique to calculate the amount of thermal growth. Once the machine is shut down, factor these thermal growth readings into the dial indicator readings and make a final alignment which allows for the actual thermal growth.

4.8 Calculating Machine Moves

Once you have taken the misalignment readings, and you have measured both the bracket sag and the thermal growth, you can then calculate the necessary moves to bring the shaft center lines into alignment. You should normally only move one of the machines.

The necessary machine moves can be calculated in a variety of ways. Some alignment systems (such as the laser or solid rod systems) come with a computer that is attached to the sensors. Once the machine dimensions and misalignment data are fed into the computer, the computer calculates the

necessary moves automatically. Other companies sell programmable calculators and/or PC-based software that performs the movement calculations.

There is also a simple graphical technique that uses a piece of grid paper to calculate machine moves. Machine dimensions are transferred to the graph paper, and the dial indicator readings are plotted above or below the shaft center lines. When all the lines are drawn the distance on the graph paper between the shaft center line of the movable machine and the center line of the fixed machine is the amount the movable machine must be moved. The graphic technique is quick, accurate, inexpensive, and it allows you to visualize the orientations of the shaft center lines in space. A graphic worksheet for shaft alignment that is used by the authors is shown at the end of this chapter, and Section 6.0 of this chapter contains an example of its use. If you want to use it for your own alignment work, please feel free to copy it as often as you like.

5.0 VIBRATION FROM MISALIGNMENT

The authors set up a large coupled test system with strain gauges mounted in the anti-friction bearing cavities, and vibration transducers mounted on all bearings of both the fixed and movable machines. The test system was designed so that shaft center line misalignment could be deliberately increased.

As the misalignment was increased, the first effect was a dramatic increase on the radial loads of the bearings of both the driver and driven machine. One of the major reasons for good alignment of shaft center lines at operating temperature is this effect on the radial bearing load.

Radial bearing loads are carefully calculated during machine design, based on known factors such as rotor weight, process loading, and dynamic belt loads. However, bearings may be running beyond their rated load because of some misalignment in the machine. Unexplained bearing failures should alert you to the possibility of shaft center line misalignment.

Following the increase in the radial bearing load, the authors noted an increase of the first order vibration, followed by an increase in the second through the sixth orders. In some cases, misalignment causes a high first order with virtually no other harmonics, while in other cases misalignment results in a lower first order but high second (and greater— often up through sixth or seventh) orders.

It is very common to analyze a vibration problem as imbalance when, in fact, it is due to shaft center line misalignment. Shaft center line misalignment is one of the major reasons that anti-friction bearings do not reach normal operating life. Misalignment is one of the major causes of premature anti-friction bearing failure.

Looking at amplitude only makes it difficult to differentiate between imbalance and misalignment. For example, an imbalance condition with some looseness or excessive clearance could result in a high first order with several harmonics, and is normally called a "classic looseness pattern." However, misaligned shafts can result in a pattern nearly identical to a classic looseness pattern.

A high vertical-to-horizontal amplitude ratio is common in machinery vibration analysis. For example, the vertical vibration at first order may be ⅓ or ¼ of the horizontal vibration amplitude at first order, or vice versa. You could assume in cases like this that this vibration is a result of resonance (horizontal to vertical amplitude ratios of 3:1 or greater are often interpreted as resonance).

However, horizontal-to-vertical amplitude ratios greater than 3:1 may also be due to a misaligned shaft. Check for resonance when these high horizontal-to-vertical amplitude ratios exist: if a resonance is not present at this frequency, then shaft center line misalignment is likely.

It is very important to carefully check the operating mode shape of the machines at the first order (at shaft frequency) when checking for shaft center line misalignment. For example, on an unbalanced

coupled machine, in most cases the phase relationship between horizontal bearings on one machine should be essentially the same (within 30 degrees or so) as the phase relationship between the vertical bearings on the same machine. This relationship applies to both the driver and the driven machine.

When shafts are misaligned, however, the horizontal bearing-to-bearing phase relationship is rarely similar to the vertical bearing-to-bearing phase relationship on either one of the machines. The inboard and outboard bearings on an unbalanced machine rarely rock precisely in phase or 180 degrees out of phase. Therefore it is very unlikely that the phase relationship between the inboard and outboard bearings of an unbalanced machine would be close to 0 or 180 degrees out of phase. This condition could exist with shaft center line misalignment.

The phase relationship between the coupling side bearings on both machines is the best evidence of shaft center line misalignment. If the phase readings on the coupling side bearing on each machine are essentially in phase or 180 degrees out of phase horizontally and vertically, it is almost certainly misalignment. This phase relationship has other causes, such as bearing pedestal resonance or a bow in the shaft. Check both of these conditions to determine whether they are responsible for the above phase relationships instead of misalignment.

It is also good to check the axial phase across the coupling, but this is not as important as taking radial phase readings both horizontally and vertically. If the axial readings are in phase or 180 degrees out of phase across the coupling, then this is a good indication of either a bent shaft or misalignment.

Note that, when you take axial readings across the coupling, the transducers will be facing in opposite directions. This movement of the transducer causes a 180-degree shift in the phase readings, which must be allowed for in analyzing the readings. So, if the transducers are opposite each other and they read 180 degrees out of phase, this really means that the axial vibration is in phase across the coupling. It is a common error when analyzing axial phase relationships to neglect the phase shift that results from opposing the transducers.

Taking axial phase readings at the 12:00, 3:00, 6:00, and 9:00 readings around a bearing face may help to determine whether the problem is a bent shaft or misalignment. If the phase at the four positions is essentially the same, the shaft is probably not bent.

6.0 A SAMPLE ANALYSIS PROBLEM

This section is an actual vibration analysis job done by the authors on a turbine-driven cooling pump at a nuclear facility. It demonstrates the techniques used to determine that misalignment was the source of the excessive vibration.

Figure 80 is a frequency domain spectra taken on the cooling pump. This spectrum display utilizes a harmonic cursor which, when positioned at a frequency peak, shows which of the other peaks are harmonically related to that frequency. The harmonic cursor appears as a small rectangle at the top of each of the frequencies that are harmonically related. In Figure 80, the first through the sixth orders are shown with six small rectangles.

Data from the harmonic cursor is shown in the bottom right corner of Figure 80. The first order (the leftmost harmonic cursor rectangle) is at 32 Hz, which is the running speed of the pump. The second order is approximately 50% of the first order.

The authors analyzed phase readings on each machine and across the coupling, compared the vertical and horizontal amplitudes at first order, and determined that no resonance was present at this frequency. Therefore, the authors investigated the alignment of the turbine and the pump shafts.

For this application, the authors used the reverse dial indicator method. The indicator brackets were mounted on a pipe and the bracket sag was measured as 9 mils. Incidentally, the estimated sag

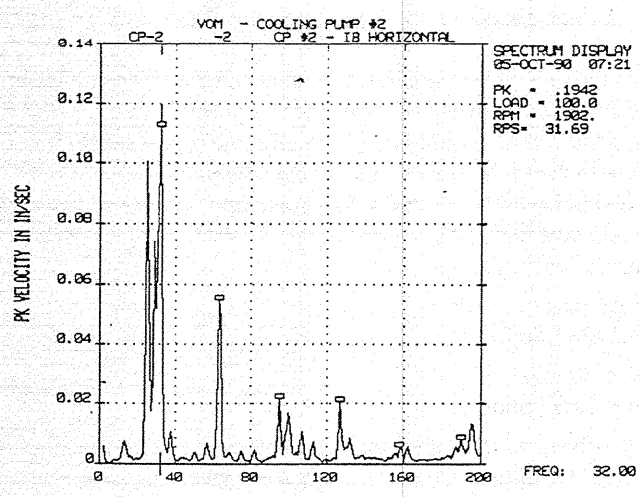


Figure 80 - *vibration spectra of cooling pump before alignment*

as found by Formulas 7-1 and 7-2 was 8.13 mils, which demonstrates that sag estimates, though they may be close, are never as accurate as actual sag measurements. Several dimensions are needed for the alignment calculations:

- the distance S1 from the indicator button position of the shim (moveable) machine to the first foot
- the distance S2 from the first foot of the shim machine to the second foot
- the distance F1 from the rim of the fixed machine to the first foot
- the distance F2 from the first foot of the fixed machine to the second foot
- the length of the dial indicator rod
- the outer diameter of the indicator rod
- the inner diameter of the indicator rod
- the weight of the indicator rod (including the dial indicator itself)

The rod length, OD, ID, and weight should be the same for both brackets. These dimensions are recorded on an alignment form such as the one shown at the end of this chapter. Both dial indicators were zeroed at the 12:00 position, and reverse dial indicator readings were then taken on both the shim and fixed machines. These readings are shown in Table 2.

Shim Machine				Fixed Machine			
Top	Bottom	Left	Right	Top	Bottom	Left	Right
.000	77.000	55.000	19.000	.000	-74.000	-42.000	-30.000

Table 2 - *dial indicator readings on shim and fixed machine*

The dial indicator sag, foot and bracket dimensions, and dial indicator dimensions were input into an alignment program developed by the authors. This program calculated both the cold shaft center

line offset and the movements necessary to bring both the fixed and moveable machines into alignment.

The program calculates movement for both the moveable and the fixed machines. It is often good to calculate moves for both machines, because it may not be possible to move the machine that you defined as the movable machine at the beginning of the alignment procedure: it may be bolt-bound, or it may need to be lowered when it is already against the sole plate.

In this case, you have no choice but to move the fixed machine. So rather than making it necessary to redefine the machines and take new data, the program calculates moves for both.

Shaft center line offset was calculated as

- vertical offset = 43 mils
  vertical angle = 1.783 minutes
- horizontal offset = 18 mils
  horizontal angle = 2.037 minutes

Note the high vertical offset, which indicates that shaft center line misalignment is indeed a problem. The movements to the shim and fixed machines are given in Table 3.

Shim Machine Movements (mils)				Fixed Machine Movements (mils)			
Vertical foot 1	Horizontal foot 1	Vertical foot 2	Horizontal foot 2	Vertical foot 1	Horizontal foot 1	Vertical foot 2	Horizontal foot 2
51.037	27.185	73.463	52.815	-29.907	-3.037	-5.019	25.407

Table 3 - *machine movements necessary for cold alignment*

The movements calculated in Table 3 are only for cold alignment, and do not take thermal growth into account. Thermal growth was measured using inside micrometers and tooling balls as discussed previously in Section 4.7 of this chapter. The thermal growth measurements are given in Table 4.

Shim Machine Growth (mils)				Fixed Machine Growth (mils)			
Vertical foot 1	Horizontal foot 1	Vertical foot 2	Horizontal foot 2	Vertical foot 1	Horizontal foot 1	Vertical foot 2	Horizontal foot 2
-2.000	-1.000	-2.000	-1.000	40.000	-10.000	49.000	-10.000

Table 4 - *measured thermal growth*

The new machine movements for the fixed and moveable machines were then recalculated, taking thermal growth into account. These new readings are shown in Table 5.

Shim Machine Movements (mils)				Fixed Machine Movements (mils)			
Vertical foot 1	Horizontal foot 1	Vertical foot 2	Horizontal foot 2	Vertical foot 1	Horizontal foot 1	Vertical foot 2	Horizontal foot 2
85.396	18.185	99.713	43.815	-71.907	5.963	-56.019	34.407

Table 5 - *machine movements necessary for hot alignment*

Notice that there is quite a difference between the cold movements shown in Table 3 and the hot movements shown in Table 5. This demonstrates the importance of accurately measuring thermal

growth and factoring it into your alignment procedure. Once the appropriate shims were made, the machines were aligned and brought back up to normal operating frequency, temperature, and load. The vibration spectra taken after alignment is shown in Figure 81. Note that the first order vibration has dropped from .11 in/sec to .01 in/sec, which is well within the vibration tolerance for this system.

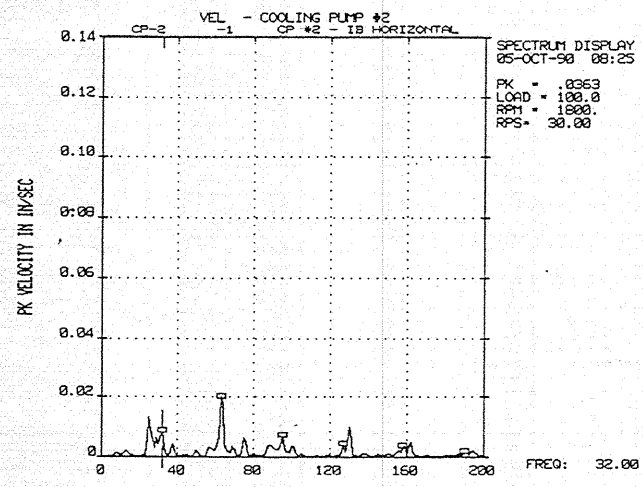


Figure 81 - *vibration spectra after alignment*

This alignment problem could also have been solved graphically as shown in Figure 82. A line is drawn at the left side of the graph denoting the center line of the fixed machine shaft. The end of this shaft is labeled A. The distance L is the length of the dial indicator rod, which in our case was 20.25 inches. So an appropriate scale is chosen for the graph (in this case the horizontal distance between two tick marks is one inch), and the location B is plotted 20.25 inches away from A.

The length S1 is the distance from point B to the first foot on the shim machine. This distance was measured as 15.5 inches and is plotted on the graph. Finally, distance S2 is the distance from the first foot on the shim machine to the second foot. This distance was measured as 43.25 inches and is plotted on the graph.

Following the instructions on the top of the chart, the bottom dial indicator reading of -74 mils taken at point A (the fixed machine) is entered on the chart. The measured sag of 9 mils is entered, the two values are added, the result of -65 is divided by 2, and a point is plotted on the graph -32.5 mils below the axis of the fixed shaft at position A. The point is plotted below the axis of the fixed machine, since the number is negative Note that the vertical scale on the graph can be different than the horizontal scale. Here we have chosen a vertical scale where the distance between two tick marks equals 2 mils.

The same procedure is followed for the readings taken off the shim machine at point B. The reading of 77 mils plus 9 for bracket sag are added together, and the result of 86 is divided by two to get 43. This value is multiplied by -1 to bring the readings "in phase" with each other (since the dial indicators are opposed), and a point is plotted -43 mils below the center line of the fixed machine at point B.

These two points are connected with a straightedge. This line now represents the center line of the shaft in the shim machine, and it is plain to see that a significant amount of angular misalignment

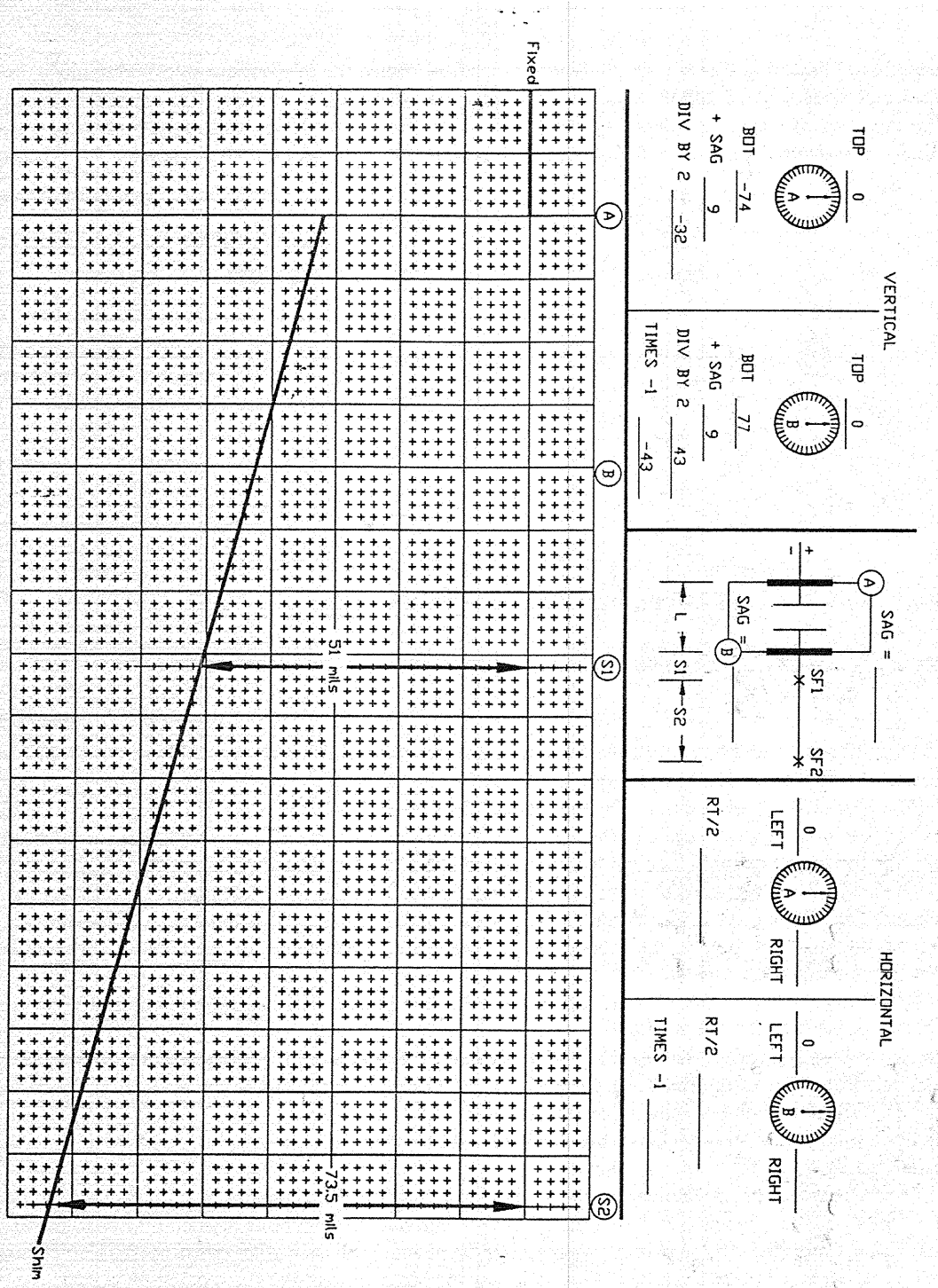


Figure 82 - *the graphic solution for cold alignment*

exists between these two machines. To find the necessary moves for the shim machine, the vertical distance between the center line of the fixed shaft and the center line of the shim shaft is measured at points S1 and S2. These lengths are measured according to the vertical scale of the graph, and the shim machine is moved this amount at both feet.

You can see that the values of 51 and 73.5 mils are very close to the cold alignment moves calculated by the program in Table 3. When thermal growth values are known, these are added to or subtracted from these measurements.

SUMMARY

The alignment of shaft center lines on coupled machines (sometimes called coupling alignment) is one of the most important aspects of proper machine installation. A popular fallacy that has existed for many years is that flexible couplings will compensate for any reasonable amount of shaft center line misalignment. Although there are generally accepted standards for shaft misalignment with various types of flexible couplings, it is best to have the shaft center lines aligned as closely as possible.

There are cases where some slight misalignment is required, such as within a gear coupling. However, it is important to have the shafts of coupled machines aligned very closely under operating conditions of temperature and load.

Shaft center line misalignment may be one of two types: parallel (sometimes called offset) or angular. Angular misalignment occurs when the shaft center lines meet at an angle. Parallel misalignment occurs when the shaft center lines are parallel, but are offset from each other. The center lines may be offset horizontally, vertically, or a combination of the two. In fact, almost all shaft center line misalignment problems are a combination of both parallel and angular misalignment.

The coupling should **not** be disassembled when using dial indicators to read misalignment. If the shafts are uncoupled, end float can cause invalid dial indicator readings. Eccentricity of a coupling hub will also corrupt dial indicator readings if the shafts are uncoupled.

The two most popular dial indicator methods for aligning shafts are the face and rim method and the reverse dial method. The reverse dial method is more accurate since it does not require long shafts in order to obtain an accurate reading.

In some cases, misalignment causes a high first order with virtually no other harmonics, while in other cases misalignment results in a lower first order but high second (and greater— often up through sixth or seventh) orders.

The phase relationship between bearings is the best way to distinguish misalignment from unbalance. If the horizontal phase relationship between the bearings on either the driver or driven machine are either in phase or 180 degrees out of phase, this almost certainly indicate misalignment. The same phase relationship is true in the vertical direction.

Phase across the coupling is the best evidence of misalignment. The phase at the inboard bearings on either side of the coupling are either in phase or 180 degrees out of phase, both in the vertical and horizontal direction.

Bearing pedestal resonance and a bent shaft can also cause these same phase relationships.

An alignment form and graphical alignment calculation sheet are given at the end of this chapter. Please feel free to copy them as often as you like.

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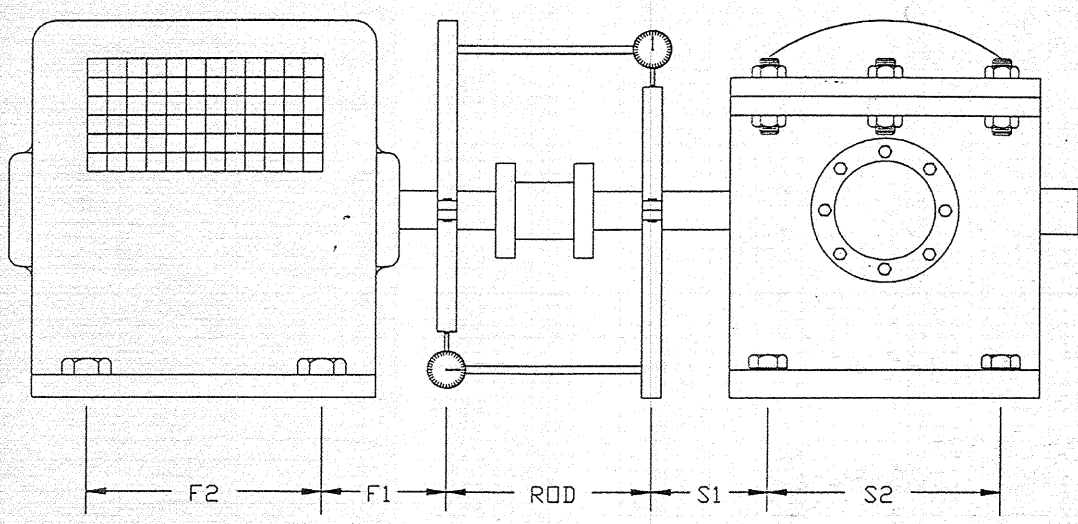
VERTICAL
TOP
0
A
BOT
+ SAG
DIV BY 2
TOP
0
B
BOT
+ SAG
DIV BY 2
TIMES -1
SAG =
SAG =
L
S1
S2
SF1
SF2
HORIZONTAL
0
LEFT
A
RIGHT
RT/2
0
LEFT
B
RIGHT
RT/2
TIMES -1

ALIGNMENT FORM RD-1
REVERSE DIAL INDICATOR

CUST _____
DATE _____



FIXED SIDE SHIM SIDE

		VERTICAL GROWTH		
		HORIZONTAL GROWTH		
		SAG		
		DIAL INDICATOR READINGS		
		TOP		
		BOTTOM		
		LEFT		
		RIGHT		
		CORRECTION MOVEMENTS		
		VERTICAL		
		HORIZONTAL		

A APPENDIX A - APPROXIMATE HARDWARE WEIGHTS

Appendix A contains tables listing calculated weights of both coarse thread (UNC) and fine thread (UNF) bolts. These tables are very useful in any balancing procedure when bolts are used to add balance weight.

Weights have been calculated for the following bolt hardware:

- Hex head cap screws
- Socket head cap screws
- Machine screws
- Socket head set screws

All weights are in ounces. Where applicable, the weight of nuts, flat washers, lock washers, and the bolt head are also given.

Table 6 - UNC Hex Head Cap Screws

BOLT DIAMETER (inches)

BOLT LENGTH (inches)	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3
HEAD	.110326	.183555	.270743	.405062	.654755	1.26257	2.14338	3.53732	5.1926	7.29617	9.99109	13.3147	19.001	29.394	42.1898	58.592	81.6853	109.705	142.23
1/4	.150324	.24815	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3/8	.170323	.280447	.412565	.599055	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1/2	.190322	.312744	.459839	.66372	.997187	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5/8	.210321	.345041	.507113	.728384	1.0828	1.93973	—	—	—	—	—	—	—	—	—	—	—	—	—
3/4	.23032	.377339	.554387	.793048	1.1684	2.07517	3.33359	—	—	—	—	—	—	—	—	—	—	—	—
7/8	.256742	.409707	.601661	.857713	1.25401	2.2106	3.53196	5.44455	—	—	—	—	—	—	—	—	—	—	—
1	.283164	.45086	.648935	.922377	1.33962	2.34603	3.73032	5.71701	8.0477	10.8995	—	—	—	—	—	—	—	—	—
1 1/4	.336009	.533165	.767835	1.0679	1.51084	2.6169	4.12706	6.26193	8.76147	11.8004	15.6523	20.0979	—	—	—	—	—	—	—
1 1/2	.388853	.615471	.886735	1.22937	1.72221	2.88776	4.52379	6.80686	9.47525	12.7012	16.7845	21.4546	28.8266	—	—	—	—	—	—
1 3/4	.441698	.697776	1.00564	1.39084	1.93359	3.21804	4.92053	7.35178	10.189	13.602	17.9167	22.8112	30.4643	44.9286	—	—	—	—	—
2	.494542	.780082	1.12454	1.5523	2.14497	3.54832	5.39613	7.8967	10.9028	14.5029	19.049	24.1679	32.1019	47.1478	65.4828	—	—	—	—
2 1/4	.547387	.862387	1.24344	1.71377	2.35635	3.8786	5.87173	8.54405	11.6166	15.4037	20.1812	25.5245	33.7395	49.3671	68.3945	92.3328	—	—	—
2 1/2	.600231	.944693	1.36234	1.87524	2.56773	4.20887	6.34733	9.19139	12.4621	16.3046	21.3134	26.8812	35.3771	51.5863	71.3061	96.0818	127.887	—	—
2 3/4	.653076	1.027	1.48124	2.0367	2.7791	4.53915	6.82293	9.83874	13.3076	17.3747	22.4457	28.2378	37.0147	53.8055	74.2177	99.8307	132.507	171.979	—
3	.70592	1.1093	1.60014	2.19817	2.99048	4.86943	7.29853	10.4861	14.1531	18.4448	23.7668	29.5945	38.6523	56.0248	77.1293	103.58	137.128	177.64	223.927
3 1/4	.758765	1.19161	1.71904	2.35964	3.20186	5.19971	7.77413	11.1334	14.9986	19.5149	25.0879	31.193	40.2899	58.244	80.041	107.329	141.748	183.302	230.735
3 1/2	.811609	1.27391	1.83794	2.5211	3.41324	5.52999	8.24973	11.7808	15.8441	20.585	26.409	32.7916	42.1923	60.4632	82.9526	111.078	146.368	188.963	237.543
3 3/4	.864454	1.35622	1.95684	2.68257	3.62461	5.86026	8.72533	12.4281	16.6896	21.6551	27.7301	34.3901	44.0947	62.6825	85.8642	114.827	150.988	194.624	244.351
4	.917298	1.43853	2.07574	2.84404	3.83599	6.19054	9.20093	13.0755	17.5352	22.7252	29.0512	35.9887	45.9971	65.2719	88.7758	118.576	155.608	200.285	251.159

NOTE: All weights shown are in ounces. To convert to grams, multiply the weight in ounces by 28.3495

Weights given in table are for steel hardware.
To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 6 - UNC Hex Head Cap Screws (cont.)

BOLT DIAMETER (inches)

BOLT LENGTH (inches)	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3
4 1/4	.970143	1.52083	2.19464	3.0055	4.04737	6.52082	9.67653	13.7228	18.3807	23.7953	30.3723	37.5872	47.8995	67.8612	91.6874	122.325	160.229	205.947	257.967
4 1/2	—	—	2.31354	3.16697	4.25875	6.8511	10.1521	14.3702	19.2262	24.8654	31.6935	39.1858	49.8019	70.4506	95.0695	126.074	164.849	211.608	264.775
4 3/4	—	—	—	—	4.47013	7.18138	10.6277	15.0175	20.0717	25.9355	33.0146	40.7843	51.7043	73.04	98.4515	129.823	169.469	217.269	271.583
5	—	—	—	—	—	7.51165	11.1033	15.6648	20.9172	27.0056	34.3357	42.3828	53.6067	75.6294	101.834	134.103	174.089	222.93	278.391
5 1/4	—	—	—	—	—	—	11.5789	16.3122	21.7627	28.0757	35.6568	43.9814	55.5091	78.2188	105.216	138.383	178.709	228.592	285.199
5 1/2	—	—	—	—	—	—	—	16.9595	22.6082	29.1458	36.9779	45.5799	57.4115	80.8081	108.598	142.664	183.994	234.253	292.007
5 3/4	—	—	—	—	—	—	—	—	23.4537	30.2159	38.299	47.1785	59.3139	83.3975	111.98	146.944	189.278	239.914	298.815
6	—	—	—	—	—	—	—	—	—	—	39.6201	48.777	61.2163	85.9869	115.362	151.225	194.563	246.308	305.623
6 1/4	—	—	—	—	—	—	—	—	—	—	—	—	63.1187	88.5763	118.744	155.505	199.847	252.703	312.431
6 1/2	—	—	—	—	—	—	—	—	—	—	—	—	—	91.1657	122.126	159.785	205.132	259.097	320.041
6 3/4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	125.508	164.066	210.416	265.491	327.651
7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	168.346	215.701	271.885	335.26
7 1/4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	220.985	278.279	342.87
7 1/2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	284.674	350.479
7 3/4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	358.089
NUT	.113852	.168808	.243719	.329794	.578448	1.12977	1.90154	3.01961	4.52216	6.50612	8.43751	11.3566	17.815	27.8401	41.1983	59.3507	79.8028	109.118	133.774
FLAT WASHER	.0971398	.17563	.227098	.368822	.581148	1.17463	1.67391	2.33655	2.86139	3.35178	3.97151	5.06575	5.85609	7.60815	9.58875	12.6041	14.5922	16.921	19.6961
LOCK WASHER	.0360038	.0636421	.100054	.149216	.209855	.395177	.645929	.988577	1.43373	1.98645	2.65973	3.47359	4.44669	7.28966	8.07892	10.3923	11.3058	12.2193	13.1328
THREAD /INCH	.159992	.258378	.378192	.517315	.684865	1.08346	1.58694	2.17969	2.8551	3.60336	4.52894	5.42659	6.55045	8.87694	11.6465	14.9959	18.4808	22.6451	27.2322

NOTE: All weights shown are in ounces. To convert to grams, multiply the weight in ounces by 28.3495

Weights given in table are for steel hardware. To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 7 - UNC Socket Head Cap Screws

BOLT DIAMETER (inches)

BOLT LENGTH (inches)	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3
HEAD	.0911595	.168978	.290383	.453217	.742896	1.39312	2.36579	3.62989	5.94316	8.27709	12.0432	15.7179	21.2862	32.7128	47.9496	66.7278	98.9781	126.507	162.286
1/4	.131158	.233571	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3/8	.151157	.265868	.432205	.647211	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1/2	.171158	.298166	.479479	.711875	1.08533	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5/8	.191155	.330463	.526753	.776539	1.17094	2.07029	—	—	—	—	—	—	—	—	—	—	—	—	—
3/4	.211154	.36276	.574027	.841204	1.25654	2.20572	3.55599	—	—	—	—	—	—	—	—	—	—	—	—
7/8	.237576	.395128	.621301	.905868	1.34215	2.34115	3.75436	5.53711	—	—	—	—	—	—	—	—	—	—	—
1	.263998	.436281	.668575	.970533	1.42776	2.47658	3.95273	5.80958	8.79827	11.8804	—	—	—	—	—	—	—	—	—
1 1/4	.316842	.518586	.787475	1.11606	1.59898	2.74745	4.34948	6.3545	9.51204	12.7813	17.7044	22.5011	—	—	—	—	—	—	—
1 1/2	.369687	.600892	.906376	1.27753	1.81035	3.01831	4.7462	6.89942	10.2258	13.6821	18.8366	23.8578	31.1119	—	—	—	—	—	—
1 3/4	.422531	.683197	1.02528	1.43899	2.02173	3.34859	5.14293	7.44434	10.9396	14.583	19.9689	25.2144	32.7495	48.2475	—	—	—	—	—
2	.475376	.765503	1.14418	1.60046	2.23311	3.67887	5.61854	7.98926	11.6534	15.4838	21.1011	26.5711	34.3871	50.4667	71.2426	—	—	—	—
2 1/4	.52822	.847808	1.26308	1.76193	2.44449	4.00915	6.09414	8.63661	12.3671	16.3846	22.2333	27.9277	36.0247	52.686	74.1542	100.469	—	—	—
2 1/2	.581065	.930114	1.38198	1.92339	2.65587	4.33943	6.56974	9.28395	13.2127	17.2855	23.3656	29.2844	37.6623	54.9052	77.0658	104.218	143.18	—	—
2 3/4	.633909	1.01242	1.50088	2.08486	2.86724	4.6697	7.04534	9.9313	14.0582	18.3556	24.4978	30.641	39.2999	57.1244	79.9774	107.967	147.8	188.781	—
3	.686754	1.09472	1.61978	2.24632	3.07862	4.99998	7.52094	10.5786	14.9037	19.4257	25.8189	31.9977	40.9376	59.3437	82.889	111.716	152.42	194.443	243.982
3 1/4	.739598	1.17703	1.73868	2.40779	3.29	5.33026	7.99654	11.226	15.7492	20.4958	27.14	33.5962	42.5752	61.5629	85.8007	115.465	157.041	200.104	250.79
3 1/2	.792443	1.25934	1.85758	2.56926	3.50138	5.66054	8.47214	11.8733	16.5947	21.5659	28.4612	35.1947	44.4776	63.7821	88.7123	119.213	161.661	205.765	257.598
3 3/4	.845287	1.34164	1.97648	2.73072	3.71276	5.99082	8.94774	12.5207	17.4402	22.636	29.7823	36.7933	46.38	66.0014	91.6239	122.962	166.281	211.426	264.406
4	.898132	1.42395	2.09538	2.89219	3.92413	6.32109	9.42334	13.168	18.2857	23.7061	31.1034	38.3918	48.2824	68.5907	94.5355	126.711	170.901	217.088	271.214

Weights given in table are for steel hardware.
To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 7 - UNC Socket Head Cap Screws (cont.)

BOLT DIAMETER (inches)

BOLT LENGTH (inches)	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3
4 1/4	.950976	1.50625	2.21428	3.05366	4.13551	6.65137	9.89894	13.8154	19.1312	24.7762	32.4245	39.9904	50.1848	71.1801	97.4472	130.46	175.521	222.749	278.022
4 1/2	—	—	2.33318	3.21512	4.34689	6.98165	10.3745	14.4627	19.9768	25.8483	33.7456	41.5889	52.0872	73.7695	100.829	134.209	180.142	228.41	284.83
4 3/4	—	—	—	—	4.55827	7.31193	10.8501	15.1101	20.8223	26.9164	35.0667	43.1875	53.9896	76.3589	104.211	137.958	184.762	234.072	291.838
5	—	—	—	—	—	7.64221	11.3257	15.7574	21.6678	27.9865	36.3878	44.786	55.892	78.9483	107.593	142.239	189.382	239.733	298.446
5 1/4	—	—	—	—	—	—	11.8013	16.4047	22.5133	29.0566	37.7089	46.3846	57.7944	81.5376	110.975	146.519	194.002	245.394	305.254
5 1/2	—	—	—	—	—	—	—	17.0521	23.3588	30.1267	39.03	47.9831	59.6968	84.127	114.357	150.8	199.287	251.055	312.062
5 3/4	—	—	—	—	—	—	—	—	24.2043	31.1968	40.3512	49.5817	61.5992	86.7184	117.739	155.08	204.571	256.717	31887
6	—	—	—	—	—	—	—	—	—	—	41.6723	51.1802	63.5016	89.3058	121.121	159.36	209.856	263.111	325.679
6 1/4	—	—	—	—	—	—	—	—	—	—	—	—	65.404	91.8952	124.504	163.641	215.14	269.505	332.487
6 1/2	—	—	—	—	—	—	—	—	—	—	—	—	—	94.4845	127.886	167.921	220.424	275.899	340.096
6 3/4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	131.268	172.202	225.709	282.293	347.706
7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	176.482	230.993	288.688	355.315
7 1/4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	236.278	295.082	362.925
7 1/2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	301.476	370.535
7 3/4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	378.144
NUT	.113852	.168808	.243719	.329794	.578448	1.12977	1.90154	3.01961	4.52216	6.50612	8.43751	11.3566	17.815	27.8401	41.1983	59.3507	79.8028	109.118	133.774
FLAT WASHER	.0971398	.17563	.227098	.368822	.581148	1.17463	1.67391	2.33655	2.86139	3.35178	3.97151	5.06575	5.85609	7.60815	9.58875	12.6041	14.5922	16.921	19.6961
LOCK WASHER	.0360038	.0636421	.100054	.149216	.209855	.395177	.645929	.988577	1.43373	1.98645	2.65973	3.47359	4.44669	7.28966	8.07892	10.3923	11.3058	12.2193	13.1328
THREAD /INCH	.159992	.258378	.378192	.517315	.684865	1.08346	1.58694	2.17969	2.8551	3.60336	4.52894	5.42659	6.55045	8.87694	11.6465	14.9959	18.4808	22.6451	27.2322

NOTE: All weights shown are in ounces. To convert to grams, multiply the weight in ounces by 28.3495

Weights given in table are for steel hardware. To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 8 - UNC Machine Screws

MACHINE SCREW SIZE

BOLT LENGHT (inches)	4		6		8		10	
	ROUND HEAD	SOCKET HEAD	ROUND HEAD	SOCKET HEAD	ROUND HEAD	SOCKET HEAD	ROUND HEAD	SOCKET HEAD
HEAD WEIGHT	.00643477	.0100188	.0119232	.0194111	.0197122	.0317552	.0305391	.0506217
1/4	.014603	.018187	.0242529	.0317407	.0380907	.0501338	.0541569	.0742395
3/8	.0186871	.0222711	.0304177	.0379056	.04728	.059323	.0659658	.0860484
1/2	.0227712	.0263553	.0365825	.0440704	.0564692	.0685123	.0777747	.0978573
5/8	.0268554	.0304394	.0427473	.0502352	.0656585	.0777016	.0895836	.109666
3/4	.0309395	.0345235	.0489122	.0564	.0748478	.0868908	.101392	.121475
1	.0391077	.0426917	.0612418	.0687297	.0932263	.105269	.12501	.145093
1 1/4	.047276	.05086	.0735714	.0810593	.111605	.123648	.148628	.168711
1 1/2	.0554442	.0590282	.0859011	.0933889	.129983	.142026	.172246	.192329
1 3/4	.0636124	.0671965	.0982307	.105719	.148362	.160405	.195864	.215946
2	.0717807	.0753647	.11056	.118048	.16674	.178784	.219482	.239564
NUT	.0192591		.034631		.0464942		.0542052	
FLAT WASHER	.00931943		.0203072		.0271764		.0352815	
LOCK WASHER	.00269291		.0045821		.00811825		.0119856	
THREAD/INCH	.0326729		.0493184		.0735139		.0944709	

Weights given in table are for steel hardware.
To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 9 - UNC Socket Head Set Screws

SET SCREW DIAMETER (inches)

BOLT LENGTH (inches)	4	6	8	10	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2	1 3/4	2
1/4	.0073019	.0108151	.0157015	.0193829	.0327567	.0504234	.0700657	.0904031	.113126	.160106	.266049	.312422	—	—	—	—	—	—	—
3/8	.0111859	.0166778	.0244408	.0306132	.0527769	.0827549	.11739	.155136	.198827	.29292	.399074	.468633	—	—	—	—	—	—	—
1/2	.0150699	.0225406	.0331796	.0418435	.0727971	.115086	.164714	.219869	.284526	.428496	.597652	.624844	.839086	1.21361	1.5385	1.9878	2.22984	2.57855	3.9648
5/8	.0189539	.0284034	.0419186	.0530738	.0928173	.147418	.212038	.284602	.370224	.564072	.79623	.897593	1.1226	1.59076	1.92313	2.48475	2.78731	3.22319	4.956
3/4	.0228379	.0342661	.0506578	.0643041	.112837	.179749	.259362	.349335	.455923	.699648	.994808	1.17034	1.47987	2.04168	2.48984	3.1638	3.34477	3.86783	5.94719
7/8	.0267219	.0401289	.0593967	.0755345	.132858	.212081	.306686	.414068	.541622	.835224	1.19339	1.44309	1.83714	2.49255	3.05656	3.84284	4.16444	4.51247	6.93839
1	.0306059	.0459917	.0681357	.0867648	.152878	.244412	.35401	.478801	.627321	.9708	1.39196	1.71584	2.1944	2.94345	3.62328	4.52188	4.98411	5.1571	7.92959
1 1/4	.038374	.0577172	.0856137	.109225	.192918	.309075	.448658	.608267	.798718	1.24195	1.78912	2.26134	2.90893	3.84524	4.75671	5.87997	6.62346	7.37869	10.8443
1 1/2	.046142	.0694427	.103092	.131686	.232959	.373738	.543307	.737732	.970116	1.5131	2.18627	2.80684	3.62347	4.74704	5.89015	7.23805	8.26281	9.60028	13.759
1 3/4	.05391	.0811683	.12057	.154147	.272999	.438401	.637955	.867198	1.14151	1.78426	2.58343	3.35234	4.338	5.64883	7.02358	8.59614	9.90215	11.8219	16.6737
2	.061678	.0928938	.138048	.176607	.313039	.503064	.732603	.996664	1.31291	2.05541	2.98059	3.89784	5.05253	6.55063	8.15702	9.95423	11.5415	14.0435	19.5884
2 1/4	.0694461	.104619	.155526	.199068	.35308	.567727	.827251	1.12613	1.48431	2.32656	3.37774	4.44334	5.76706	7.45242	9.29045	11.3123	13.1808	16.265	22.5031
2 1/2	.0772141	.116345	.173004	.221528	.39312	.63239	.921899	1.2556	1.65571	2.59771	3.7749	4.98884	6.48159	8.35421	10.4239	12.6704	14.8202	18.4866	25.4178
2 3/4	.0849821	.12807	.190482	.243989	.43316	.697053	1.01655	1.38506	1.8271	2.86886	4.17205	5.53434	7.19613	9.25601	11.5573	14.0285	16.4595	20.7082	28.3326
3	.0927501	.139796	.20796	.26645	.473201	.761716	1.1112	1.51453	1.9985	3.14001	4.56921	6.07984	7.91068	10.1578	12.6908	15.3866	18.0989	22.9298	31.2473
3 1/4	.100518	.151521	.225438	.28891	.513241	.826379	1.20584	1.64399	2.1699	3.41117	4.96636	6.62534	8.62519	11.0596	13.8242	16.7447	19.7382	25.1514	34.162
3 1/2	.108286	.163247	.242916	.311371	.553282	.891042	1.30049	1.77346	2.3413	3.68232	5.36352	7.17084	9.33972	11.9614	14.9576	18.1027	21.3776	27.373	37.0767
3 3/4	.116054	.174973	.260394	.333832	.593322	.955705	1.39514	1.90293	2.51269	3.95347	5.76068	7.71634	10.0543	12.8632	16.0911	19.4608	23.0169	29.5946	39.9914
4	.123822	.186698	.277872	.356292	.633362	1.02037	1.48979	2.03239	2.68409	4.22462	6.15783	8.26184	10.7688	13.765	17.2245	20.8189	24.6563	31.8161	42.9061
4 1/4	.13159	.198424	.29535	.378753	.673403	1.08503	1.58444	2.16186	2.85549	4.49577	6.55499	8.80734	11.4833	14.6668	18.3579	22.177	26.2956	34.0377	45.8208
4 1/2	—	—	—	—	—	—	—	—	—	—	—	—	12.1978	15.5686	19.4914	23.5351	27.935	36.2593	48.7355
4 3/4	—	—	—	—	—	—	—	—	—	—	—	—	12.9124	16.4704	20.6248	24.8932	29.5743	38.4809	51.6502

NOTE: All weights shown are in ounces. To convert to grams, multiply the weight in ounces by 28.3495

Weights given in table are for steel hardware. To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 10 - UNF Hex Head Cap Screws

BOLT DIAMETER (inches)

BOLT LENGTH (inches)	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
HEAD	.110326	.183555	.270743	.405062	.654755	1.26257	2.14338	3.53732	5.1926	7.29617	9.99109	13.3147	19.001
1/4	.153818	.252425	—	—	—	—	—	—	—	—	—	—	—
3/8	.175563	.28686	.424247	.61309	—	—	—	—	—	—	—	—	—
1/2	.197309	.321294	.475415	.682432	1.02434	—	—	—	—	—	—	—	—
5/8	.219055	.355729	.526583	.751775	1.11673	1.99564	—	—	—	—	—	—	—
3/4	.240801	.390164	.577751	.821117	1.20913	2.14225	3.41989	—	—	—	—	—	—
7/8	.267223	.424653	.628919	.89046	1.30153	2.28886	3.63264	5.56911	—	—	—	—	—
1	.293645	.465805	.680087	.959802	1.39392	2.43548	3.84539	5.85936	8.21861	11.1748	—	—	—
1 1/4	.34649	.548111	.798987	1.10997	1.57871	2.7287	4.27089	6.43987	8.97511	12.1445	16.0372	20.6908	—
1 1/2	.399334	.630416	.917887	1.27144	1.79009	3.02193	4.6964	7.02038	9.73161	13.1141	17.2465	22.1681	29.6069
1 3/4	.452179	.712722	1.03679	1.4329	2.00147	3.35221	5.1219	7.60089	10.4881	14.0838	18.4557	23.6413	31.3745
2	.505023	.795027	1.15569	1.59437	2.21285	3.68249	5.5975	8.1814	11.2446	15.0534	19.6649	25.1165	33.1422
2 1/4	.557868	.877333	1.27459	1.75584	2.42423	4.01276	6.0731	8.82875	12.0011	16.0231	20.8742	26.5918	34.9098
2 1/2	.610712	.959638	1.39349	1.9173	2.6356	4.34304	6.5487	9.47609	12.8466	16.9927	22.0834	28.067	36.6775
2 3/4	.663557	1.04194	1.51239	2.07877	2.84698	4.67332	7.0243	10.1234	13.6921	18.0628	23.2926	29.5422	38.4451
3	.716401	1.12425	1.63129	2.24024	3.05836	5.0036	7.4999	10.7708	14.5377	19.1329	24.6137	31.0175	40.2128
3 1/4	.769246	1.20655	1.75019	2.4017	3.26974	5.33388	7.9755	11.4181	15.3832	20.203	25.9349	32.616	41.9804
3 1/2	.82209	1.28886	1.86909	2.56317	3.48112	5.66415	8.4511	12.0655	16.2287	21.2731	27.256	34.2145	43.8828
3 3/4	.874935	1.37117	1.98799	2.72463	3.69249	5.99443	8.9267	12.7128	17.0742	22.3432	28.5771	35.8131	45.7852
4	.927779	1.45347	2.10689	2.8861	3.90387	6.32471	9.4023	13.3602	17.9197	23.4133	29.8982	37.4116	47.6876

Weights given in table are for steel hardware.
To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 10 - UNF Hex Head Cap Screws (cont.)

BOLT DIAMETER (inches)

BOLT LENGTH (inches)	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
4 1/4	.980624	1.53578	2.22579	3.04757	4.11525	6.65499	9.8779	14.0075	18.7652	24.4834	31.2193	39.0102	49.59
4 1/2	—	—	2.34469	3.20903	4.32663	6.98527	10.3535	14.6549	19.6107	25.5535	32.5404	40.6087	51.4924
4 3/4	—	—	—	—	4.53801	7.31554	10.8291	15.3022	20.4562	26.6236	33.8615	42.2073	53.3948
5	—	—	—	—	—	7.64582	11.3047	15.9495	21.3017	27.6937	35.1826	43.8058	55.2972
5 1/4	—	—	—	—	—	—	11.7803	16.5969	22.1473	28.7638	36.5038	45.4044	57.1996
5 1/2	—	—	—	—	—	—	—	17.2442	22.9928	29.8339	37.8249	47.0029	59.102
5 3/4	—	—	—	—	—	—	—	—	23.8383	30.904	39.146	48.6015	61.0044
6	—	—	—	—	—	—	—	—	—	—	40.4671	50.2	62.9068
6 1/4	—	—	—	—	—	—	—	—	—	—	—	—	64.8092
NUT	.110848	.164277	.233594	.315759	.554826	1.08102	1.82674	2.91284	4.37518	6.23774	8.11105	10.8064	17.0478
FLAT WASHER	.0971398	.17563	.227098	.368822	.581148	1.17463	1.67391	2.33655	2.86139	3.35178	3.97151	5.06575	5.85609
LOCK WASHER	.0360038	.0636421	.100054	.149216	.209855	.395177	.645929	.988577	1.43373	1.98645	2.65973	3.47359	4.44669
THREAD /INCH	.173967	.275478	.409344	.55474	.739168	1.1729	1.70201	2.32204	3.02601	3.87862	4.83693	5.90092	7.0706

NOTE: All weights shown are in ounces. To convert to grams, multiply the weight in ounces by 28.3495

Weights given in table are for steel hardware. To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 11 - UNF Socket Head Cap Screws

BOLT DIAMETER (inches)

BOLT LENGTH (inches)	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
HEAD	.0911595	.168976	.290383	.453217	.742896	1.39312	2.36579	3.62989	5.94316	8.27709	12.0432	15.7179	21.2662
1/4	.134651	.237846	—	—	—	—	—	—	—	—	—	—	—
3/8	.156397	.272281	.443887	.661245	—	—	—	—	—	—	—	—	—
1/2	.178143	.306716	.495055	.730587	1.11248	—	—	—	—	—	—	—	—
5/8	.199889	.34115	.546224	.79993	1.20488	2.12619	—	—	—	—	—	—	—
3/4	.221634	.375585	.597392	.869273	1.29727	2.2728	3.6423	—	—	—	—	—	—
7/8	.248057	.410074	.64856	.938615	1.38967	2.41942	3.85505	5.66167	—	—	—	—	—
1	.274479	.451227	.699728	1.00796	1.48206	2.56603	4.0678	5.95193	8.96917	12.1557	—	—	—
1 1/4	.327323	.533532	.818628	1.15812	1.66686	2.85926	4.4933	6.53244	9.72568	13.1254	18.0894	23.094	—
1 1/2	.380168	.615838	.937528	1.31959	1.87823	3.15248	4.9188	7.11295	10.4822	14.095	19.2986	24.5693	31.8921
1 3/4	.433012	.698143	1.05643	1.48106	2.08961	3.48276	5.3443	7.69346	11.2387	15.0647	20.5079	26.0445	33.6598
2	.485857	.780449	1.17533	1.64252	2.30099	3.81304	5.81991	8.27397	11.9952	16.0343	21.7171	27.5197	35.4274
2 1/4	.538701	.862754	1.29423	1.80399	2.51237	4.14332	6.29551	8.92131	12.7517	17.004	22.9263	28.9949	37.1951
2 1/2	.591546	.94506	1.41313	1.96546	2.72374	4.47359	6.77111	9.56866	13.5972	17.9736	24.1355	30.4702	38.9627
2 3/4	.64439	1.02737	1.53203	2.12692	2.93512	4.80387	7.24671	10.216	14.4427	19.0437	25.3448	31.9454	40.7304
3	.697235	1.10967	1.65093	2.28839	3.1465	5.13415	7.72231	10.8633	15.2882	20.1138	26.6659	33.4206	42.498
3 1/4	.750079	1.19198	1.76983	2.44986	3.35788	5.46443	8.19791	11.5107	16.1337	21.1839	27.987	35.0192	44.2657
3 1/2	.802924	1.27428	1.88873	2.61132	3.56926	5.79471	8.67351	12.158	16.9792	22.254	29.3081	36.6177	46.1681
3 3/4	.855768	1.35659	2.00763	2.77279	3.78063	6.12498	9.14911	12.8054	17.8248	23.3241	30.6292	38.2163	48.0705
4	.908613	1.43889	2.12653	2.93426	3.99201	6.45526	9.62471	13.4527	18.6703	24.3942	31.9503	39.8148	49.9729

Weights given in table are for steel hardware.
To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 11 - UNF Socket Head Cap Screws (cont.)

BOLT DIAMETER (inches)

BOLT LENGTH (inches)	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
4 1/4	.961457	1.5212	2.24543	3.09572	4.20339	6.78554	10.1003	14.1001	19.5158	25.4843	33.2715	41.4134	51.8753
4 1/2	—	—	2.36433	3.25719	4.41477	7.11582	10.5759	14.7474	20.3613	26.5344	34.5926	43.0119	53.7777
4 3/4	—	—	—	—	4.62615	7.4461	11.0515	15.3948	21.2068	27.6046	35.9137	44.6104	55.6801
5	—	—	—	—	—	7.77637	11.5271	16.0421	22.0523	28.6747	37.2348	46.209	57.5825
5 1/4	—	—	—	—	—	—	12.0027	16.6894	22.8978	29.7448	38.5559	47.8075	59.4849
5 1/2	—	—	—	—	—	—	—	17.3368	23.7433	30.8149	39.877	49.4061	61.3873
5 3/4	—	—	—	—	—	—	—	—	24.5888	31.885	41.1981	51.0046	63.2897
6	—	—	—	—	—	—	—	—	—	—	42.5192	52.6032	65.1921
6 1/4	—	—	—	—	—	—	—	—	—	—	—	—	67.0945
NUT	.110848	.164277	.233594	.315759	.554826	1.08102	1.82674	2.91284	4.37518	6.23774	8.11105	10.8064	17.0478
FLAT WASHER	.0971398	.17563	.227098	.368822	.581148	1.17463	1.67391	2.33655	2.86139	3.35178	3.97151	5.06575	5.85609
LOCK WASHER	.0360038	.0636421	.100054	.149216	.209855	.395177	.645929	.988577	1.43373	1.98645	2.65973	3.47359	4.44669
THREAD /INCH	.173967	.275478	.409344	.55474	.739168	1.1729	1.70201	2.32204	3.02601	3.87862	4.83693	5.90092	7.0706

NOTE: All weights shown are in ounces. To convert to grams, multiply the weight in ounces by 28.3495

Weights given in table are for steel hardware. To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 12 - UNF Machine Screws

MACHINE SCREW SIZE

BOLT LENGTH (inches)	4		6		8		10	
	ROUND HEAD	SOCKET HEAD	ROUND HEAD	SOCKET HEAD	ROUND HEAD	SOCKET HEAD	ROUND HEAD	SOCKET HEAD
HEAD WEIGHT	.00643477	.0100188	.0119232	.0194111	.0197122	.0317552	.0305391	.0506217
1/4	.0150699	.0186539	.0251268	.0326147	.0386837	.0507268	.0561698	.0762524
3/8	.0193875	.0229715	.0317286	.0392165	.0481695	.0602126	.0689852	.0890678
1/2	.0237051	.0272891	.0383304	.0458183	.0576553	.0696983	.0818005	.101883
5/8	.0280226	.0316067	.0449322	.0524201	.0671411	.0791841	.0948159	.114699
3/4	.0323402	.0359242	.051534	.0590219	.0766269	.0886699	.107431	.127514
1	.0409754	.0445594	.0647376	.0722255	.0955984	.107641	.133062	.153145
1 1/4	.0496105	.0531945	.0779412	.085429	.11457	.126613	.158693	.178775
1 1/2	.0582457	.0618297	.0911448	.0986326	.133542	.145585	.184323	.204406
1 3/4	.0668808	.0704648	.104348	.111836	.152513	.164556	.209954	.230037
2	.075516	.0791	.117552	.12504	.171485	.183528	.235585	.255668
NUT	.0190835		.0342535		.0462		.0532068	
FLAT WASHER	.00931943		.0203072		.0271764		.0352815	
LOCK WASHER	.00269291		.0045821		.00811825		.0119856	
THREAD/INCH	.0345405		.0528142		.075886		.102523	

Weights given in table are for steel hardware.
To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

Table 13 - UNF Socket Head Set Screws

SET SCREW DIAMETER (inches)

BOLT LENGTH (inches)	4	6	8	10	1/4	5/16	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8	1 1/2
1/4	.0077459	.0116462	.0162655	.0212972	.036254	.054703	.0778621	.0997693	.126718	.202491	.294847	.348047	—	—	—	—	—
3/8	.0118519	.0179245	.0252865	.0334847	.0580229	.0891743	.129084	.169185	.219212	.326498	.442271	.522071	—	—	—	—	—
1/2	.015958	.0242029	.0343075	.0456721	.0797918	.123648	.180307	.238601	.311706	.473266	.655247	.696094	.92463	1.35139	1.69266	2.22522	2.4902
5/8	.020064	.0304812	.0433285	.0578596	.101561	.158117	.231529	.308017	.4042	.620035	.868224	.986657	1.22953	1.76298	2.11582	2.78152	3.11274
3/4	.02417	.0367595	.0523495	.0700471	.123329	.192588	.282751	.377433	.496694	.766803	1.0812	1.27722	1.60819	2.24832	2.72108	3.51992	3.73529
7/8	.0282761	.0430379	.0613706	.0822345	.145098	.227059	.333973	.446849	.589188	.913572	1.29418	1.56778	1.98684	2.73367	3.32633	4.25831	4.62006
1	.0323821	.0493162	.0703916	.094422	.166867	.261531	.385195	.516265	.681681	1.06034	1.50715	1.85834	2.36549	3.21901	3.93159	4.99671	5.50482
1 1/4	.0405941	.0618728	.0884336	.118797	.210405	.330473	.48764	.655097	.866669	1.35388	1.93311	2.43947	3.12279	4.18969	5.1421	6.4735	7.27434
1 1/2	.0488062	.0744295	.106476	.143172	.253943	.399416	.590084	.793929	1.05166	1.64741	2.35908	3.02059	3.8801	5.16037	6.35262	7.9503	9.04386
1 3/4	.0570183	.0869862	.124518	.167547	.29748	.468358	.692529	.932761	1.23664	1.94095	2.78501	3.60172	4.6374	6.13106	7.56313	9.42709	10.8134
2	.0652303	.0995428	.14256	.191922	.341018	.537301	.794973	1.07159	1.42163	2.23449	3.21097	4.18284	5.3947	7.10174	8.77364	10.9039	12.5829
2 1/4	.0734424	.112099	.160602	.216297	.384556	.606244	.897418	1.21043	1.60662	2.52802	3.63692	4.76397	6.15201	8.07242	9.98415	12.3807	14.3524
2 1/2	.0816544	.124656	.178644	.240672	.428094	.675186	.999862	1.34926	1.79161	2.82156	4.06287	5.34509	6.90931	9.04311	11.1947	13.8575	16.122
2 3/4	.0898665	.137213	.196686	.265046	.471631	.744129	1.10231	1.48809	1.9766	3.1151	4.48883	5.92622	7.66662	10.0138	12.4052	15.3343	17.8915
3	.0980786	.149769	.214728	.289421	.515169	.813071	1.20475	1.62692	2.16158	3.40864	4.91478	6.50734	8.42392	10.9845	13.6157	16.8111	19.661
3 1/4	.106291	.162326	.23277	.313796	.558707	.882014	1.3072	1.76575	2.34657	3.70217	5.34073	7.08847	9.18122	11.9552	14.8262	18.2878	21.4305
3 1/2	.114503	.174883	.250812	.338171	.602244	.950957	1.40964	1.90459	2.53156	3.99571	5.76669	7.66959	9.93853	12.9258	16.0367	19.7646	23.2
3 3/4	.122715	.187439	.268854	.362546	.645782	1.0199	1.51208	2.04342	2.71655	4.28925	6.19264	8.25072	10.6958	13.8965	17.2472	21.2414	24.9696
4	.130927	.199996	.286896	.386921	.68932	1.08884	1.61453	2.18225	2.90153	4.58278	6.61859	8.83184	11.4531	14.8672	18.4577	22.7182	26.7391
4 1/4	.139139	.212553	.304938	.411296	.732858	1.15778	1.71697	2.32108	3.08652	4.87632	7.04455	9.41297	12.2104	15.8379	19.6683	24.195	28.5086
4 1/2	—	—	—	—	—	—	—	—	—	—	—	—	12.9677	16.8086	20.8788	25.6718	30.2781
4 3/4	—	—	—	—	—	—	—	—	—	—	—	—	13.725	17.7793	22.0893	27.1486	32.0477

NOTE: All weights shown are in ounces. To convert to grams, multiply the weight in ounces by 28.3495

Weights given in table are for steel hardware. To find weights for hardware of other materials:

for stainless steel, multiply weight by .9880
for aluminum, multiply weight by .3442
for brass, multiply weight by 1.0766

GLOSSARY OF TERMS

acceleration - Change in velocity with respect to time.

accelerometer - A transducer that measures acceleration (also see **transducer**).

amplification range - The range of operating frequencies where the vibratory motion due to the forcing function is amplified by the system.

amplitude - The distance of a mass from its at rest position. When vibratory motion is plotted as a sine wave, the peak amplitude is the maximum distance of the wave from the zero line.

analog analyzer - An electronic vibration analyzer whose output is proportional (or ANALOGous) to its input. Analog analyzers display the amount of vibration on an analog meter.

attenuation - The decrease in strength of an electrical signal, usually as a result of the distance the signal travels. As electrical signals and vibratory waves travel, they decrease in strength the farther they travel.

axial plane - The plane in which the axis of a shaft or bearing lies.

bandwidth - The range of frequencies that a transducer or analyzer can accommodate. For filters, bandwidth specifies the smallest frequency difference that can be deteced (this is also called **resolution**). Also see **constant percentage bandwidth filter**.

bearing misalignment - A misalignment condition that results when the bearings supporting a shaft are not aligned with each other. The bearings may be mounted in different planes, cocked relative to the shaft, or distorted due to foundation settling or thermal growth.

beat frequency - A frequency that results when two or more primary frequencies are "beat" together (i.e. are present at the same time). The sum and difference of these primary frequencies create new frequencies called beat frequencies.

cavitation - A condition that occurs when a local pressure drop inside a pump allows cavities filled with fluid vapor to form. These cavities collapse when they contact the high pressure side of the pump, causing both noise and vibration. This same condition in fans and blowers is caused **starvation**.

chord - A line segment that joins two points on a curve or circle.

circular frequency - The amount of revolution in a unit time.

coherence - The linear relationship between two signals. Coherence is calculated over several averages by multiplying individual frequency values by their corresponding output values.

constant percentage bandwidth filter - A filter whose frequency resolution increases as the frequency to which it is tuned decreases. These filters are used in analog analyzers to tune in to specific vibration components.

contact angle - The angle at which a rolling element in an anti-friction bearing contacts the bearing races. Changes in bearing load often change the contact angle.

Coulomb damping - Another way to refer to vibratory damping (also see **damping**).

CPS - Cycles per second (also called **Hertz**).

crest factor - The peak value of a wave divided by the wave's RMS value. Pure sine waves have a crest factor of 1.414. Some analog analyzers only display a crest factor reading instead of a time waveform. Comparing these readings to 1.414 allows you to estimate the nature of the time waveform.

critical speed range - see **amplification range**

critically damped - A condition where a system's damped resonant frequency is equal to zero. Critically damped systems will move through one cycle when excited, but they do not vibrate (also see **damping**).

cross effect - The resultant vibration in an imbalance plane (or planes) caused by the addition of a trial weight in another plane.

cross-channel analysis - Using an analyzer with two or more input channels to calculate such functions as phase, coherence, and transfer function.

cycle - The time separation between two identical conditions of motion (also see **period**).

damping - The dissipation of vibratory energy. Damping can occur from friction, viscosity, and internal hysteresis.

degree of freedom - The number of axes along which a system is free to move.

differentiation - The mathematical process used to convert displacement into velocity, or velocity into acceleration. Differentiation is the reverse of integration.

digital analyzer - An analyzer that converts analog vibration information into a digital format suitable for computers. Digital analyzers allow precise, real-time analysis of vibration.

displacement - The total amount of movement of a vibrating system, equal to two times amplitude.

double differentiation - Performing differentiation twice to convert displacement into acceleration (also see **differentiation**).

double integration - Performing integration twice to convert acceleration into displacement (also see **integration**).

dynamic range - The ratio of the smallest amplitude to the largest amplitude that an analyzer can detect.

fault frequency - A frequency generated by a fault in an anti-friction bearing.

fixed machine - The machine whose position is not changed during shaft alignment. Compare with **shim machine**.

forced vibration - Vibration that is maintained from cycle to cycle by a forcing function (also see **forcing function**).

forcing function - A periodic force that causes a system to vibrate, and then maintains that vibration.

Fourier transform - The process of converting a time domain display into its various frequency components.

free space amplitude - The amplitude at which no amplification due to resonance occurs.

free vibration - Vibration where no energy is added after the initial force that begins the vibration.

frequency - The number of complete cycles in unit time.

frequency analysis - Machine analysis performed by examining a frequency domain display. The frequencies that are present are used to determine the vibration forcing function.

frequency domain - A display of all the frequencies of vibration that are present in a sample.

friction damping - Damping that occurs when two parts of a system slide against each other, like piston rings against a cylinder wall (also see **damping**).

harmonic - A frequency that is an integer multiple of a given frequency.

harmonic distortion - Distortion of a given wave occurring from the presence of one or more harmonics of that wave (also see **harmonic**).

hydraulic instability - A vibratory condition where the lubricant in a sleeve bearing no longer flows as a fluid film. Caused primarily when the shaft moves off-center due to load, eccentricity, or unbalance, hydraulic instability leads to oil whirl and oil whip.

hysteresis damping - Damping that occurs due to internal friction created when the molecules of the material slide against each other (also see **damping**).

Hz (Hertz) - the number of cycles per second of a wave.

influence coefficients - A set of numbers that define how changes made in one imbalance plane affect other imbalance planes in the system.

integration - The mathematical process used to convert acceleration into velocity, or velocity into displacement. Integration is the reverse of differentiation.

least squares balancing - A balancing method used to find a compromise balance solution. This method is used in systems where a satisfactory correction in one plane inevitably results in high vibration in other planes. By using least squares balancing, acceptable vibration readings are found in all planes.

mass - A body of matter that occupies a certain volume of three-dimensional space.

misalignment - A condition where the center lines of two coupled shafts are not exactly in line with each other. The two center lines may be displaced parallel to each other (called **parallel misalignment**) or they may intersect at an angle (called **angular misalignment**).

multiple plane balancing - A balancing method where a rotor is "divided up" into several imbalance planes. Correction weights are calculated and added in each plane.

natural frequency - The frequency at which a system will vibrate when excited by a single input force. The frequency of a system's free vibration.

non-synchronous components - Vibration frequencies that are not an integer multiple of the reference signal.

offset misalignment - A misalignment condition where the shaft center lines are displaced parallel to each other and do not intersect. This condition is also called **parallel misalignment**.

oil whirl - A condition of hydraulic instability where the shaft inside a sleeve bearing rides on the unstable vortex created by the shaft being moved off center. Oil whirl is proportional to shaft frequency.

oil whip - A resonant condition that occurs when the oil whirl frequency coincides with the first critical of the shaft (also see **oil whirl**).

overdamped - A condition where a system's damped resonant frequency is imaginary. This condition arises when g/l in Formula 2-3 is less than $(\zeta_{2m})^2$. Overdamped systems exhibit non-oscillatory motion (also see **damping**).

parallel misalignment - See **offset misalignment**.

peak - The value of a wave's maximum amplitude.

period - The time distance between two identical conditions of motion of a wave.

phase - The time separation between two events.

phase shift - Any change in phase that occurs as a result of the system response.

phase angle - The time separation between two events when time is measured in degrees (also see **phase**).

radial plane - The plane in which the radius of a shaft or bearing lies.

radian - An angular measurement where the length of a subtended arc segment is equal to the radius of the circle on which it lies.

resonance - The condition where the forcing function frequency coincides with a system's resonant frequency.

resonant frequency - The frequency at which a system will amplify the energy input from the forcing function, causing a dramatic (and often disastrous) rise in vibration amplitude.

restoring force - the backwards pull (or push) of a spring against the deflecting force (also see **spring constant**).

resultant - The vector obtained when two or more vectors are added or subtracted. When as-is and trial weight imbalance vectors are added, the vibration that results is the resultant vibration.

RMS - A measure of the power contained in a wave. The RMS of a pure sine wave is equal to .707 times the peak value of the wave.

shim machine - The machine whose position is changed during shaft alignment. Compare with **fixed machine**.

signal-to-noise ratio - The ratio of a wave's peak amplitude to that of the wave's noise floor (the **noise floor** is the level of noise that is present even if no signal is present; this level is the quietest that the wave can be).

sine wave - A wave whose periodic behavior exactly matches the plot of the sine function.

single plane balancing - A balancing method that assumes that a rotor's imbalance is located in a single plane. Correction weights are added in this plane. Also see **multiple plane balancing**.

sinusoidal - Having the shape of a sine wave.

spring - Any part of a system that supports mass and behaves like a spring (that is, the part pushes or pulls against a force).

spring constant - the number of pounds necessary to extend a spring (or deflect a structural member) one inch. The spring constant is expressed in lbs/in, and is written mathematically by the letter *k*.

spring-mass system - A mechanical system containing a spring, a mass, and damping. Mechanical systems treated for vibration are typical spring-mass systems.

starvation - A condition that occurs when a local pressure drop inside a fan allows small vacuum cavities to form. These cavities collapse when they contact the high pressure side of the fan, causing both noise and vibration. This same condition in pumps is caused **cavitation**.

strobe light - A bright, rapidly flashing light used to "freeze" a rotating shaft, thus allowing you to read phase from the apparent still image.

sub-synchronous components - Vibration frequencies that are less than 1*RPM of the shaft.

synchronous components - Vibration frequencies that occur at integer multiples of shaft RPM. Synchronous components are also called **phase locked components**.

system - A mechanical assembly that consists of a mass, a spring, and damping.

thermal growth - A change in length of a system component caused by expansion due to heating.

time domain - A display of a system's vibratory behavior during a specific period of time.

transducer - A device used to convert vibratory motion into an electrical signal proportional to that motion. The signal may be proportional to the displacement, the velocity, or the acceleration.

transfer function - A frequency domain description of how the system acts on one signal to produce another signal.

trial weight - A weight used in single- and multi-plane balancing to determine the system's response characteristics.

tribo effect - The induction of noise in a shielded cable as a result of the cable being bent.

underdamped - A condition where a system's damped resonant frequency is greater than zero, arising when g/l in Formula 2-3 is greater than $(c/2m)^2$. Underdamped systems vibrate through several cycles before coming to rest (also see **damping**).

vector - A mathematical quantity having a magnitude (length) and a direction. Vibration forces are represented as vectors.

velocity - The change in displacement with respect to time.

vibration - The complex, periodic motion of a rotating or reciprocating system in response to one or more forcing functions.

vibration signature - The measure of all the frequencies comprising a system's vibratory movement.

viscous damping - Damping caused by the resistive force of a viscous liquid against a moving body. Most processes in industry are viscous processes (also see **damping**).

weight - The acceleration of a mass by gravity.

wiping - The rubbing of a shaft against the bearing material in a sleeve bearing.

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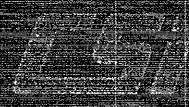
The *Simplified Handbook of Vibration Analysis* series explains how to determine the health of rotating/reciprocating machinery by analyzing vibration signatures. Written in a non-technical, easy-to-follow manner, these books emphasize practical experience and real-world problem solving.

Although some mathematical equations are included in the text, a background in physics or mathematics is not necessary. The emphasis is on *how* to measure and resolve vibration problems, not on *why* the vibrations behave the way they do. These texts will be an invaluable resource for any maintenance personnel responsible for machinery vibration analysis.

Volume 1 begins by discussing how mechanical systems vibrate in response to forcing functions. This leads to a discussion of vector representation of force, creating a foundation for the analysis techniques discussed later in the handbook. The various types of transducers that can be used to collect vibration data are explained in detail, along with strengths and weaknesses of each type. A chapter on General Analysis Techniques discusses the vibration analysis procedure *itself*, offering many useful hints on identifying the types of vibration that could be present in your machine. Finally, chapters on Phase, Field Balancing, and Shaft Alignment apply the analysis procedure to balancing rotors in the field and to aligning coupled shafts.



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